QG Height Tendency Equation

1. Traditional form

A diagnostic expression for geopotential height tendency is derived by combining the QG vorticity and thermodynamic equations. The resulting equation for frictionless, adiabatic flow is known as the QG height tendency equation [eq. 5.6.13 in Bluestein (1992), p. 330]

$$\left(\nabla_p^2 + \frac{f_0}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi = f_0\left[-\boldsymbol{V_g}\cdot\boldsymbol{\nabla_p}(\zeta_g + f)\right] - \frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[\frac{R_d}{p}\left(-\boldsymbol{V_g}\cdot\boldsymbol{\nabla_p}T\right)\right]$$

where χ is the local time tendency of geopotential, $\boldsymbol{\nabla_p}$ is the gradient operator on a pressure surface, ∇_p^2 is the Laplacian operator on a pressure surface, $\boldsymbol{V_g}$ is the geostrophic wind, ζ_g is the geostrophic relative vorticity, R_d is the gas constant for dry air, p is the pressure, f is the Coriolis parameter, f_0 is the Coriolis constant ($10^{-4}\ s^{-1}$), σ is the static stability parameter (assumed constant at $2.0\times10^{-6}\ m^2Pa^{-2}s^{-2}$), and T is the temperature.

The right-hand-side (RHS) is calculated to determine the forcing for geopotential height changes at the 500 hPa level. Positive values indicate forcing for geopotential height decreases. Negative values indicate forcing for geopotential height increases. Note that this diagnostic calculation results in the forcing for geopotential height tendency not the actual geopotential height tendency because we do not solve the Laplacian on the left-hand-side.

The first term (A) on the RHS is advection of geostrophic absolute vorticity by the geostrophic wind. This term is evaluated at 500 hPa. Cyclonic vorticity advection (term A > 0) is associated with forcing for geopotential height decreases. Anticyclonic vorticity advection (term A < 0) is associated with geopotential height increases. For the example of a 500 hPa trough, term A is zero along the trough axis and is positive downstream and negative upstream of the trough axis. Hence, the geopotential height tendency is negative downstream and positive upstream of the trough axis. Troughs will move in the direction from positive to negative geopotential height tendency. Conversely, ridges will move in the direction from negative to positive geopotential height tendency. Therefore, the **absolute vorticity advection term (A) acts as the propagation mechanism for troughs and ridges**.

The second term (B) on the RHS is differential advection of temperature by the geostrophic wind. This term is evaluated using temperature advection at 700 and 300 hPa. Finite differencing is used to evaluate the vertical derivative of temperature advection. Warm air advection maximized at 700 hPa and decreasing upward to 300 hPa (term B < 0) is associated with forcing for geopotential height increases at 500 hPa. Cold air advection maximized at 700 hPa and decreasing upward to 300 hPa (term B > 0) is associated with forcing for geopotential height decreases at 500 hPa. For the example of a 500 hPa trough located upstream of the attendant surface cyclone, term B is > 0 upstream of the surface cyclone beneath the 500 hPa trough axis, resulting in forcing for geopotential height decreases along the trough axis. In this case, cold air advection weakening upward beneath a 500 hPa trough will deepen the trough. Conversely, warm advection weakening with height downstream of the surface cyclone beneath the downstream 500 hPa ridge will strengthen the ridge. Therefore,

the differential temperature advection term (B) acts as the amplification mechanism for troughs and ridges.

2. QGPV form

An alternative form of the diagnostic equation for geopotential height tendency, which results from combining and manipulating the frictionless form of the QG vorticity equation and adiabatic form of the QG thermodynamic equation, can be written using QGPV [eq. 5.8.15 in Bluestein (1992), p. 373]

$$\left(\nabla_p^2 + \frac{f_0}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi = f_0\left[-\boldsymbol{V_g} \cdot \boldsymbol{\nabla_p}(QGPV)\right]$$

where χ is the local time tendency of geopotential, $\boldsymbol{\nabla_p}$ is the gradient operator on a pressure surface, ∇_p^2 is the Laplacian operator on a pressure surface, $\boldsymbol{V_g}$ is the geostrophic wind, p is the pressure, f_0 is the Coriolis constant (10^{-4} s^{-1}), and σ is the static stability parameter (assumed constant at $2.0\times10^{-6}\ m^2Pa^{-2}s^{-2}$). The QGPV is defined as [eq. 5.8.10 in Bluestein (1992), p. 372]

$$QGPV = \zeta_g + f + \frac{\partial}{\partial p}\left(\frac{f_0}{\sigma}\frac{\partial\Phi}{\partial p}\right)$$

where ζ_g is the geostrophic relative vorticity, f is the Coriolis parameter, Φ is the geopotential, and other symbols are same as defined earlier. The geostrophic absolute vorticity term is calculated at 500 hPa. The stability term is calculated as described in Hakim et al. (1995; pp. 2665-2666).

As with the traditional form, the RHS is calculated to determine the forcing for geopotential height changes at the 500 hPa level. Positive values indicate forcing for geopotential height decreases. Negative values indicate forcing for geopotential height increases. Note that this diagnostic calculation results in the forcing for geopotential height tendency not the actual geopotential height tendency because we do not solve the Laplacian on the left-hand-side.

The term on the RHS is advection of QGPV by the geostrophic wind at 500 hPa. In the Northern Hemisphere, positive QGPV advection (term > 0) is associated with forcing for geopotential height decreases. Negative QGPV advection (term < 0) is associated with forcing for geopotential height increases. The QGPV form of the height tendency equation avoids the possibility of cancelling effects between the vorticity advection and differential thermal advection terms in the traditional form of the equation by containing only one forcing function on the RHS.

References:

Bluestein, H. B., 1992: *Principles of Kinematics and Dynamics*. Vol. I. *Synoptic-Dynamic Meteorology in Midlatitudes*. Oxford University Press, 431 pp.

Hakim, G. J., L. F. Bosart, and D. Keyser, 1995: The Ohio Valley wave-merger cyclogenesis event of 25-26 January 1978. Part I: Multiscale case study. *Mon. Wea. Rev.,* **123,** 2663-2692.

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