Petterssen's Development Equation

Following the work by R. C. Sutcliffe in the late 1940s, S. Petterssen developed an expression for the local tendency of geostrophic relative vorticity at 1000 hPa. Assuming that divergence at 500 hPa is negligible compared to that at the surface, and vorticity advection at the surface is negligible compared to that at 500 hPa, the following equation for the local tendency of 1000 hPa geostrophic relative vorticity goes as [eq. 8.10 in Carlson (1998), p. 185]

$$\frac{\partial \zeta_0}{\partial t} = -\boldsymbol{V_5} \cdot \boldsymbol{\nabla_p}(\zeta_5 + f) - \frac{g}{f_0}\left[\nabla_p^2(\bar{A}_T + \bar{S} + \bar{H})\right]$$

where t is time, $\boldsymbol{\nabla_p}$ is the gradient operator on a pressure surface, ∇_p^2 is the Laplacian operator on a pressure surface, $\boldsymbol{V_5}$ is the geostrophic wind at 500 hPa, ζ_5 is the geostrophic relative vorticity at 500 hPa, ζ_0 is the geostrophic relative vorticity at 1000 hPa, g is gravitational acceleration, f is the Coriolis parameter, and f_0 is the Coriolis constant (10^{-4} s^{-1}).

The first term (A) on the right-hand-side (RHS) is advection of 500 hPa geostrophic absolute vorticity by the 500 hPa geostrophic wind. When a trough at 500 hPa is located upstream (typically to the west) of a surface cyclone, the positive vorticity advection ahead of the trough and over the surface cyclone will contribute to **increases of 1000 hPa cyclonic vorticity over the center of the surface cyclone**.

The second term on the RHS represents the Laplacian of the thermal contributions to development: 1000-500 hPa thickness advection ($\bar{A}_T$), adiabatic influences ($\bar{S}$), and diabatic influences ($\bar{H}$). The overbars signify an average over the 1000-500 hPa layer. Specifically, these terms are written as $\bar{A}_T = -\boldsymbol{V_A} \cdot \boldsymbol{\nabla_p} h$, $\bar{S} = \frac{R_d \bar{\bar{s}} \Delta p_5}{g \bar{p}} \bar{\omega}$, and $\bar{H} = \frac{R_d \bar{\bar{s}} \Delta p_5}{g c_p \bar{p}} \bar{\dot{Q}}_{db}$, where $\boldsymbol{V_A}$ is the mean geostrophic wind in the 1000-500 hPa layer, h is the 1000-500 hPa thickness, R_d is the gas constant for dry air, $\bar{p}$ is the layer-mean pressure, Δp_5 is the pressure difference between 1000 and 500 hPa, $\bar{\omega}$ is the layer-mean vertical velocity, c_p is specific heat, $\bar{\dot{Q}}_{db}$ is layer-mean diabatic heating, and $\bar{\bar{s}}$ is layer-mean static stability $\left(s = -\frac{T}{\theta}\frac{\partial \theta}{\partial p}\right)$.

The Laplacian of thickness advection (B) results in $\frac{\partial \zeta_0}{\partial t} > 0$ in the region of maximum warm air advection and $\frac{\partial \zeta_0}{\partial t} < 0$ in the region of maximum cold air advection. Note that warm and cold advection tend to occur ahead of and behind the surface cyclone, respectively. Hence, the surface cyclone will move in the direction from negative to positive vorticity tendencies. Therefore, the **thickness advection contributes to the surface cyclone motion**.

The Laplacian of vertical velocity (C) results in $\frac{\partial \zeta_0}{\partial t} > 0$ in the region of descent (adiabatic warming) and $\frac{\partial \zeta_0}{\partial t} < 0$ in the region of ascent (adiabatic cooling) provided that the stratification is stable $\bar{\bar{s}} > 0$.

The Laplacian of diabatic heating (not shown) results in in $\frac{\partial \zeta_0}{\partial t} > 0$ in the region of maximum diabatic heating and $\frac{\partial \zeta_0}{\partial t} < 0$ in the region of maximum diabatic cooling.

Consult Carlson (1998), pp. 185-186, for an expanded discussion of Petterssen's development equation. It is also best to go to the original source: Petterssen (1956) devotes an entire chapter on interpreting the above equation (pp. 320-339).

References:

Carlson, T. N., 1998: *Mid-Latitude Weather Systems.* Amer. Meteor. Soc., 507 pp.

Petterssen, S., 1956: *Motion and Motion Systems.* Vol. I. *Weather Analysis and Forecasting.* McGraw-Hill, 428 pp.

Contact:
Dr. Thomas J. Galarneau
Department of Hydrology and Atmospheric Sciences, The University of Arizona
P.O. Box 210011, Tucson, AZ 85721-0011
Email: tgalarneau@email.arizona.edu
Web: https://tgalarneau.faculty.arizona.edu/