On the interaction of tropical-cyclone-scale vortices. II: Discrete vortex patches

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SUMMARY

The interaction of vortex patches in relation to the observed scales and features reported in Part I of this paper is investigated. It is found that the initial approach of compound vortices, such as tropical cyclones, arises from distortion of their, weaker, outer vorticity fields. Care needs to be taken in applying simple concepts, such as the propagation of each vortex on the gradient of its neighbour. For similar vortices, the initial interaction consists of mutual orbit and approach with only minor fluctuations in shape. Once the vortices approach to within a critical separation distance, rapid merger occurs. A simplified expression for this critical distance is derived and it indicates that binary tropical cyclones must approach to within 150–300 km before merger of the cores can occur. When vortices have substantial differences in size or intensity, the smaller/weaker system shears into the outer circulation of the other and no core merger occurs. It is suggested that this is typical of the interaction between tropical cyclones and both the monsoon trough and embedded mesoscale convective complexes.

The motion of interacting vortices consists largely of a mutual orbit, which scales according to their relative sizes and intensities. An important finding, however, is that higher-frequency meanders may develop. This indicates that a tropical cyclone interacting with a monsoonal shear zone may develop internal asymmetries and a trochoidal oscillation as a direct result of such interaction. Further, weak mesoscale vortices within the cyclone circulation may continue to cause a track meander long after they have been sheared beyond the resolving capacity of atmospheric observations.

1. INTRODUCTION

The mechanisms of vortex interaction have provided fertile ground for study for more than a century since Kirchoff (1876) showed that a system of interacting point vortices formed a Hamiltonian system. Point vortices consist of discrete vorticity points surrounded by potential flow, and can interact only by mutual advection. The instantaneous motion of any one vortex is given by the sum of the circulations from all other vortices in the plane (or the fluid volume for line vortices). Thompson (1883) showed that systems of more than three interacting line vortices were non-integrable, and it is now known that the interaction of multiple point vortices is chaotic and possesses a marked sensitivity to initial conditions (Aref 1983). Since the equations of motion can be decomposed into the interactions of a large number of point vortices (Sommerfeld 1964), this chaotic interaction has considerable implications for the onset of turbulence and the flow evolution in fluids. Studies that use geostrophic point vortices (Morikawa and Swenson 1971) indicate that such chaotic behaviour may be present in the motion of synoptic-scale vortices.

The best documented cases of interacting tropical-cyclone-scale vortices are those of binary tropical cyclones, also known as the Fujiwhara effect following the classical studies by Fujiwhara (1921, 1923, 1931). Fujiwhara used a series of laboratory experiments in water to show that vortices of the same rotation placed in near proximity to each other tend to approach in a spiral orbit that has the same sense of rotation as the original vortices. The end result is a merger of the vortices to produce a new vortex that is both larger and more intense than either of the original pair. Conversely, vortices of opposite rotation were observed to repel each other. Fujiwhara also quotes empirical

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and unpublished 'laws' by T. Okada that atmospheric anticyclones tend to repel cyclones and that cyclones tend to attract each other.

In Part I of this study, Lander and Holland (1993) found that several modifications to the classical Fujiwhara model are required (as shown in their Fig. 1). In this revised model, the period of interaction is characterized by a variable cyclonic rotation during which the cyclones typically converge, but may diverge or remain at a constant separation distance. Those interacting cyclones that develop far apart initially converge with no evidence of mutual interaction, and often with anticyclonic trajectories. Capture then occurs quite sharply, typically within a few hours. The interaction may end with merger of the cyclone pair, destruction of one or both partners (by moving over land, for example) or a sudden release and escape. The rapidity of the capture and release processes is indicative of nonlinear transitions between two stable regimes, one in which the cyclones interact, and the other where they move independently of each other.

Lander and Holland show that interactions between three tropical cyclones can be broken down into separate binary interactions that conform to the same basic model. Interactions also occur between tropical cyclones and other vortices in their vicinity. For example, the potential recurvature of a tropical cyclone located equatorward of a mobile mid-latitude trough can be considered in terms of vortex interaction, as can interactions with anticyclonic cells in the subtropical ridge.

Holland and Lander (1993) have related the meander of typhoon Sarah (1989) to interactions with Mesoscale Convective Complexes that developed within the cyclone circulation and suggested that many of the observed meanders in tropical-cyclone tracks could occur by this mechanism. They also documented examples of interacting swarms of mesoscale vortices within super clusters in the tropics.

This paper, together with Part III (Holland and Dietachmayer 1993), provides an initial theoretical examination of these observational findings in terms of two-dimensional interacting cyclonic vortices. We are particularly concerned with the manner in which vortices can capture each other; reasons why some vortices merge, while others escape after a period of interaction; and with the role played by divergence, spherical geometry, and the gradient in earth vorticity. We neglect the effects of vertical structure and diabatic forcing that are present in tropical cyclones. Our rationale is that an understanding of the relevant two-dimensional dynamics will provide a basis for further studies in which vertical structure and diabatic forcing are included. Although we concentrate on configurations typical of those found in tropical-cyclone situations, the results are applicable to the interaction of small vortices* in general and complement the study of large vortices by Williams and Wilson (1988).

We use contour-dynamics solutions of interacting vortex patches to examine some of the detailed interactions that occur. The following section summarizes the vortex structure and the basis of the contour-dynamics approach. The interaction of two equal cyclonic vortices is presented in section 3; in section 4 we investigate interacting vortices of different sizes and intensities, and in section 5 the motion of compound vortices is examined. Section 6 contains our conclusions.

2. Method

Vortex patches, consisting of a region of constant vorticity surrounded by a potential flow, form the simplest type of finite-scale vortex. A familiar example is the Rankine

* For this study we define small as being substantially less than the Rossby radius of deformation. This is consistent with Williams and Wilson (1988) and provides a logical means of comparison between studies of laboratory, atmospheric, oceanic, and extraterrestrial vortices.
combined vortex (Depperman 1947), which has a circular region of constant vorticity and has been used in several studies as a first-order approximation to tropical cyclones (Holland 1980). For example, Haurwitz (1951) provided a first-order theoretical extension of Fujiwhara's work by using a non-deformable Rankine vortex. He derived an equation for the speed of mutual rotation as a function of size, intensity and separation distance, which bore similarities to those that have been observed. However, Haurwitz was not able to model the observed mutual attraction, and provided no explanation for the capture/escape mode described in Part I.

(a) Vortex structure

The vortex patches used in this study consist of initially circular Rankine combined vortices of the form:

\[
\begin{align*}
\nu &= cr \\
\zeta &= 2c \\
\nu &= cR^2r^{-1} \\
\zeta &= 0
\end{align*}
\]

\[ r < R \]

\[ r \geq R \]

where \( \nu \) is the azimuthal flow, \( R \) is the radius of the vortex patch (Fig. 1(a)), \( r \) is the distance from the centre of the patch, \( \zeta \) is the vorticity at \( r \) and \( c \) is a scaling constant*.

Originally introduced to meteorology by Depperman (1947), such vortex structure has been used in several studies to approximate the core region of a tropical cyclone (Holland 1980). As Shapiro and Ooyama (1990) and Carr and Williams (1990) have shown, vortices with typical tropical-cyclone structure tend to produce an homogenized region of near constant potential vorticity in the core region with a sharp drop to low environmental values. This is illustrated in Fig. 1(b) by the radial profile of vorticity and azimuthal winds after 48 hours of integration of a vortex in the barotropic modelling study of Holland and Dietachmayer (1993). It is obvious that a vortex patch provides a good first-order approximation to the continuous profile. The system is non-dimensionalized for computation and applies to vortices of any scale. Typical values for a tropical cyclone are:

* We adopt a southern hemispheric convention, with \( c < 0 \).
$\xi_{tc} = 10^{-3} \xi_n \text{ s}^{-1}$
$R_{tc} = 50 R_n \text{ km}$
$v_{tc} = v_n \xi_{tc} R_{tc} \text{ m s}^{-1}$
$t_{tc} = \frac{1}{\xi_{tc}} \text{ s.}$

where $v$ is the azimuthal wind, and subscripts 'tc' and 'n' refer to tropical cyclone values and normalized vortex values respectively.

(b) Integration by contour dynamics

In numerical simulations of turbulent two-dimensional flows, the emergence of long-lived, isolated vorticity structures has been observed. McWilliams (1984) has shown that these structures resist the enstrophy cascade, appearing to increase in energy through encounters with other such isolated structures.

To study the interactions between vortex patches we make use of the contour-dynamics method of solution for inviscid, incompressible fluids in two dimensions, following Zabusky et al. (1979) and Overman and Zabusky (1982). This method makes use of the observation that such flows develop steep-sided finite-area-vortex-regions or vortex patches and thus can be represented by the closed contour surrounding each constant-vorticity region. The contour is approximated by discrete nodes (Fig. 2), and the method computes the dynamic interactions between the nodes directly. Thus, the dimensionality is reduced to one, and no horizontal boundary conditions are required. For a more detailed description of the model the reader is referred to the above authors.

Figure 2. An illustration of the specification of the boundary of a vortex patch with discrete nodes.

3. Two equal vortices

The interaction of two vortex patches occurs through the mutual advection of vorticity by the sheared potential flow surrounding each patch. When two equal vortices of the same sense of rotation are sufficiently far apart they rotate about each other at a constant separation distance whilst inducing transient perturbations from symmetry (Figs. 3(a) and 4(a)). As has been found in previous studies (Rossow 1977; Aref 1983; Griffiths and Hopfinger 1987; Melander et al. 1988), there is a critical separation distance at which a distinct bifurcation between merger and non-merger occurs (Fig. 5). Vortices of equal size and intensity at less than this critical separation (around $3.2 R$ for the vortices used
Figure 3. (a) Interaction of two vortex patches of equal size and vorticity started at a separation of $4R$; (b) rapid merger of the two patches when placed at an initial separation of $2.7R$.

Figure 4. Trajectories of the fluid parcel initially located at the centre of the vortex patches in Fig. 3: (a) non-merging (Fig. 3(a)), (b) merging (Fig. 3(b)).

here) merge within slightly less than one orbit, regardless of their initial separation distance. In the absence of diffusion, vortex patches at greater than this critical initial separation never merge.

Melander et al. (1988) reported that this interaction consists of three separate stages:

(i) the vortex patches orbit continuously with transient perturbations from symmetry (Fig. 3(a));

(ii) the amplitude of the perturbations becomes sufficiently large to allow the patches to touch and briefly exchange fluid before moving apart again (not shown); or
(iii) the patches eject vortex filaments, are rapidly distorted, and merge within one orbit (Fig. 3(b)).

Stage (ii) only occurs over a limited part of the interaction phase domain between the non-merging and merging solutions. Since the related conditions are unlikely to occur (or be observable) in tropical cyclones, we concentrate on stages (i) and (iii).

The merged vortex initially takes the form of a 2:1 Kirchoff ellipse (Kirchoff 1876) which continues to rotate with trailing spiral filaments (Fig. 3(b)). Notice that the original vortices are placed at opposite foci of the ellipse and retain much of their original identity. This was demonstrated in laboratory experiments by Griffiths and Hopfinger (1987). Continued filamentation draws fluid away at the extrema of the ellipse as the system relaxes back to a stable energy state (Dritschel 1986). Quite complex filament behaviour may occur (Melander et al. 1988), including interaction between filaments, reconnection to the main vortex, and roll-up of filaments to form new vortices. Waugh (1992) demonstrated that complicated regimes exist beyond those discussed here and the reader is referred to this author for a more detailed discussion. Ultimately the merged vortex returns to a circular shape surrounded by a tangle of thin filaments.

Merger produces a new vortex that has the same vorticity but is larger than either of the original systems. The resulting circulation is, therefore, much stronger and contains local maxima in the vicinity of vortex spirals (Fig. 6). These local maxima are similar to those associated with contracting eye-walls and secondary hurricane intensification (Willoughby et al. 1982). We suggest that vortex filamentation may provide an explanation for the initiation of such secondary intensification.

The tracks of the fluid parcels initially at the centre of each vortex patch (Fig. 4) are clearly quite different. Whilst the non-merging vortices orbit at a constant separation
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Figure 6. Wind-speed profiles along the indicated sections of the compound vortex (inset) that forms from the merger of two equal vortices. The dashed line indicates the wind profile of each of the original vortices.

distance, with superimposed minor oscillations (Fig. 4(a)), the merging vortices approach each other and accelerate along a spiral path (Fig. 4(b)). The spiral bands of vorticity also move entirely by advection. Thus, no inertial or gravity-wave propagation (Willoughby et al. 1984; Diercks and Anthes 1976a,b) is required to produce realistic, moving spiral bands.

4. TWO UNEQUAL VORTEX PATCHES

When two cyclonic vortices of different sizes and intensities interact, a different pattern to that observed with the identical systems emerges. Dritschel and Waugh (1992) showed that for vortices of different sizes and intensities, a far larger variety of interactions can occur. They point out that the merger of two equal vortices produces only a single vortex with occasionally two very small satellite vortices from the roll-up of filaments. However, the interaction of unequal vortices can often produce two vortices of differing size and intensity from the original vortices. We observe that one vortex often retains its identity whilst the other is sheared off.

We illustrate the features of this interaction with a series of experiments in which one patch (L) is fixed at three times the radius of the other (S), the initial separation distance is fixed at twice the large-patch radius, and the relative magnitudes, $\zeta_L:\zeta_S$ of the patches are changed (Fig. 7).

(a) Experimental results

For $\zeta_L:\zeta_S = 3:1$ (Fig. 7(a)), the smaller vortex is strongly sheared and wraps around the larger system, which experiences only minor distortion. This experiment corresponds to the imposition of a minor perturbation in the periphery of a tropical cyclone and provides an explanation of the strong tendency for symmetry in the cyclone core (McAlpin 1987; Carr and Williams 1990; Smith et al. 1990; Shapiro and Ooyama 1990). Notice that during the shearing and filamentation of the small vortex, the leading edge converges towards the larger vortex, whilst the trailing end moves outwards. This arises from the tendency for the sheared vortex to advect itself on a cyclonic rotation.
Figure 7. Interactions of a large and small vortex (size ratio 3:1) with varying intensity ratios, $\eta_1: \eta_2$: (a) 3:1, (b) 2:1, (c) 1:1 (d) 1:2, (e) 1:3. The initial separation distance is twice the radius of the larger vortex in all cases.
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Increasing the relative intensity of the smaller patch to $\zeta L : \zeta S = 2:1$ (Fig. 7(b)), then 1:1 (Fig. 7(c)), introduces increased distortion on the larger system and brings the sheared filament from the smaller vortex into direct contact with it. Physically, Fig. 7(c) represents the interactions that might occur when a small tropical cyclone moves into the vicinity of a larger system of equal intensity.

When the smaller system is twice the intensity of the larger one, marked distortion of the larger system occurs and the two vortices merge (Fig. 7(d)) in a similar manner to that described above for equal vortices. We suggest that this type of process may occur between mesoscale vortices in the early development stage of tropical cyclones. Further increasing the smaller vortex intensity to $\zeta L : \zeta S = 1:3$ (Fig. 7(e)) results in a shearing of the larger patch into a long filament that wraps around the weakly distorted smaller system. A physical analogue of this process is the interaction of a tropical cyclone with a nearby large monsoon depression, as is often observed in the western North Pacific basin. The sheared large vortex is associated with a localized wind jet that has considerable resemblance to the belt of westerly winds along and around the equatorward side of developing tropical cyclones. This indicates that the presence of such a westerly jet may be due to the interaction between the two systems, rather than a precursor to cyclone formation as is often hypothesized in forecast reports (e.g. the Annual tropical cyclone reports of the Joint Typhoon Warning Center, Guam).

(b) Quantification of merger situations

The sharp bifurcation between merger and non-merger in Fig. 5 and the associated features of the unequal vortex mergers in Fig. 7 arise from the competing effects of strain imposed on each vortex by the sheared potential flow of the other vortex, and the tendency towards rotational symmetry in each vortex patch. Since orbiting circular vortices will experience no strain under solid-body rotation, Melander et al. (1988) introduced a rotating coordinate system located at the centre of the two vortices with the same rotation rate as the vortex centres. From this perspective they were able to elucidate the major mechanisms contributing to all three stages of merger by equal vortices, and we refer the reader to their paper for details. Such an approach cannot be applied to unequal vortices, however, and the reader is referred to the paper by Dritschel and Waugh (1992) for a detailed description and quantification of an unequal vortex merger. We need to be able to indicate the likely separation at which atmospheric vortices will merge, so we adopt a simplified method based on explicit calculation of the strain and restoring processes. We neglect stage (ii) of the merger process and assume that once the strain reaches a value sufficient to produce a significant distortion of the patch, the symmetricization breaks down and rapid evolution to a new stable state occurs*.

The net strain on a circular vortex patch $(R_2, \zeta_2)$ orbiting another vortex $(R_1, \zeta_1)$ consists of the difference between the wind shear across the patch and the shear associated with solid-body rotation. Provided that the size of the patch is small compared with the separation distance, $\Delta$, from the other vortex, the ratio of strain to restoring tendency can be approximated by

$$\alpha = \left. \frac{r \partial v_1/r}{\zeta_2} \right|_{r = \Delta}$$

where $v_1$ is the potential flow surrounding the first vortex. Substituting a Rankine vortex structure into Eq. (3) and neglecting second-order terms, we arrive at an approximate

* In the example in Fig. 3, the new stable state is a merger of the vortices to form a Kirchhoff ellipse with trailing vortex filaments.
specification of the critical separation distance, $\Delta_c$, inside which the patch can be expected to be sheared into, or merged with, the other vortex:

$$\Delta_c = \frac{R_1}{\alpha_c} \left( \frac{\zeta_1}{\zeta_2} \right)^{\frac{1}{3}}$$  \hspace{1cm} (4)

where $\alpha_c$, the critical strain ratio, is a type of Richardson number that may be estimated empirically from experiments such as those in Fig. 5. We tested the predictions from Eq. (4) against experimental results for a range of vortex configurations. A value of $\alpha_c = 1/3.5$ provided the best results, and correctly predicted each of the experimental findings in Fig. 7. Considerable scatter was found amongst all the experiments conducted, however, so that $\Delta_c$ provides only an indication of the likely separation required for vortices to merge, or to become sheared.

For a tropical cyclone, we can approximate the vorticity by

$$\zeta = 2 \frac{V_m}{R_m}$$  \hspace{1cm} (5)

where $V_m$, $R_m$ are the value and radius of the maximum winds. The equivalent vortex-patch radius also is approximately $R_m$, so that Eq. (5) becomes

$$\Delta_c = \frac{R_m}{\alpha_c} \left( \frac{V_m R_m}{V_m R_m} \right)^{\frac{1}{3}}$$  \hspace{1cm} (6)

For typical parameters, Eq. (6) indicates that interacting tropical cyclones must approach to within 150–300 km separation before the merger process can commence. At this separation rapid merger, or shearing destruction of one of the systems, can be expected. The mutual approach at larger distances requires some other mechanism, as is discussed in section 5 and in Part III.

(c) Vortex motion

The tracks of the fluid parcel initially at the centre of the large vortex (Fig. 8(a)) display some interesting characteristics that depend on the nature of the interaction between the two vortices. When the smaller vortex is much weaker than the larger vortex (left trajectory in Fig. 8(a), which corresponds to Fig. 7(a)), the centre of the larger vortex is perturbed in a small, localized trajectory, which changes only gradually in amplitude. During this time the smaller system is sheared into a long spiral surrounding the larger vortex. The amplitude of the perturbation on the larger vortex centre increases with increased intensity of the smaller system (moving left to right in Fig. 8(a)), as does the degree of complexity in the resultant trajectory.

We indicate the scale of oscillation in spatial coordinates related to tropical cyclones in Fig. 8(b), which has an imposed advecting flow of approximately 5 m s$^{-1}$ from the north-east to simulate interaction of atmospheric vortices in a steering current. Scaling the larger vortex in Fig. 7(a) to a radius equivalent to the radius of maximum winds of a tropical cyclone, say 30 km, and a vorticity of 10 $f$ (where $f$ is the Coriolis parameter), then the small vortex is equivalent to a mesoscale feature of radius 10 km and vorticity 3.3 $f$, just outside the radius of maximum winds. The resulting oscillation (Fig. 8(b)) has an amplitude around 10 km, wavelength 50 km and period 6 hours. Alternatively, if we scale the larger vortex to be a monsoon depression of equivalent patch radius 200 km and a vorticity of $f$, and the smaller system to be a developing tropical cyclone of radius 50 km and vorticity $3f$ (Fig. 7(e)), the tropical cyclone oscillates with a period of 10 hours, amplitude of 15 km and wavelength of 120 km (Fig. 8(b)).
These results support the findings of Holland and Lander (1993) that many of the observed meanders in tropical-cyclone tracks arise from interactions with mesoscale systems that develop within the cyclone circulation. In particular, our results in Figs. 7(a), 7(b) and 7(c) support their observational and barotropic modelling analysis of the impact of mesoscale vortices within the circulation of typhoon Sarah on its motion.

The tracks in Fig. 8 also indicate that the induced meander of a tropical cyclone by a mesoscale vortex may continue long after the initial system becomes unresolvable by the available observational networks. Notice in particular that the smaller vortex continues to cause a distinct meander of the larger system well after it has been reduced to a thin filament (Fig. 7(a)). An important further conclusion is that several scales of meander may be induced during vortex interaction. The non-merging vortices in Fig. 4(b) experience a
small-scale oscillation superimposed on the overall orbital configuration, and all trajectories in Figs. 8(a) and 8(b) contain high-frequency oscillations. For example, the small vortex in Fig. 7(e) loops around in a circular orbit as it wraps up the larger vortex, but it is also distorted into an elliptical shape which rotates at a higher frequency. This experiment indicates that a cyclone interacting with the monsoon shear zone could be induced to undergo small-scale trochoidal oscillations.

5. COMPOUND VORTICES

The first-order effects of a gradient of vorticity surrounding each vortex can be obtained by introducing piecewise continuous vortices made up of a series of concentric vortex patches. The simplest configuration is the pair of interacting compound vortices shown in Fig. 9 in which a strong core is surrounded by a substantially weaker region of cyclonic vorticity. The outer section of each vortex immediately becomes distorted and shears along and around the other vortex, and the two vortices approach each other. This initial approach is driven by the distortion of the outer region vortices. The core vortices were originally located at a radius of $4R$, which we have shown to be too large for merger of independent vortex patches. The cores retain a quasi-symmetric shape until they approach to within around three times their initial radius. At this stage rapid merger occurs in a close analogy to that described for single patches in section 4(b).

Buntine and Pullin (1989) demonstrate that the merger of an isolated pair of equal Burgers vortices follow much the same pattern. In their case studies they show that at low Reynolds numbers ($Re \leq 40$) merger is controlled by viscosity and strain effects with little interaction with the vorticity field. At larger Reynolds numbers ($Re \geq 160$) the vorticity interactions play a more active role in the dynamics. Regions of weak vorticity are left behind the faster rotating cores, forming the spiral arms which are quickly diffused by viscous effects. Eventually the system returns to an axisymmetric equilibrium solution. At very high Reynolds numbers ($Re = 640, 1280$), the thinning of the spiral arms at their shoulders (Fig. 9, $t = 45$) is attributed to the effect of the arms being thrown off much faster than the vorticity can diffuse. In this case the vorticity in the arms is nearly conserved in the presence of diffusion. Thus as the arms extend they become thinner at the shoulder where the extension occurs.

The interaction of a small vortex patch with a larger compound vortex containing the same intensity core is shown in Fig. 10. The compound vortex has vorticity ratios of $-1: -0.1: +0.1$ between the core and outermost patch, and simulates actual tropical
cyclones in which the outer region vorticity becomes anticyclonic. In this case the single patch strips outer region vorticity from the larger system, and after sufficient time this shearing advection process develops a new compound vortex structure around the smaller vortex. Note that little distortion of the cores occurs, as is expected from Eq. (4), and the presence of the anticyclonic outer vorticity in the compound system results in a net divergence of the two centres.

Figure 10. Interaction of a simple compound vortex and a single vortex patch. The ratio of vorticities for the compound system is -1.0:-0.1:+0.1, and the centres are initially placed 5R apart.

A vortex patch placed inside a compound vortex with cyclonic gradient directed inwards (Fig. 11(a); vorticities -1:-0.3:-0.1:-1) induces a rapid inward advection of external fluid into close proximity to the compound vortex core. At the same time relatively-cyclonic fluid is drawn away from near the core into the outer region of the compound vortex. This is accompanied by strong shearing of the single patch, which retains some structure as a component of a long spiral of vortex filaments. Parcels initially located at the vortex centres move along a cyclonic orbit at nearly constant separation. Introducing an additional outer segment to the compound vortex that has stronger cyclonic vorticity (Fig. 11(b)) induces a similar interaction sequence. We note that the mutual approach of the two cores in Fig. 11(b) is stronger than that observed for Fig. 11(a), even though the single vortex patch is located in a region of anticyclonic vorticity gradient directed towards the compound vortex core.

Maintaining the same compound structure as in Fig. 11(b), but changing the vorticity to produce an anticyclonic outer region, whilst maintaining a cyclonic gradient toward the core of the same magnitude of that in Fig. 11(a), induces the anticyclonic orbit with net divergence of the vortex cores shown in Fig. 11(c). In this case the vortices also retain distinct identities and there is no inward penetration of external fluid to the compound vortex core. Filamentation of the inner cyclonic vortex segment is observed, however.

We have shown that the interaction of two vortices depends on substantially more than the sign of the vorticity gradient from each vortex. We have seen that the causative mechanism is the distortion of the outer structure of each vortex, so that the evolving fields develop considerable structure. Although the tracks in Figs. 9 and 10 support the conclusions of DeMaria and Chan (1984) that an inward directed cyclonic (anticyclonic) vorticity gradient will cause interacting tropical cyclones to approach (move away from) each other, local variations in vorticity gradient, and the absolute magnitude of the vorticity field, can dominate the interaction. In real-world situations quite complex structures could develop. Thus, markedly different results are to be expected from an intuitive extrapolation of the larger-scale findings of, for example, Fiorino and Elsberry (1989), Evans et al. (1991) and Holland and Evans (1992). These experiments support similar findings by Smith et al. (1990) that distortion of the background fields can lead to nonlinear changes in the motion of barotropic vortices.
Figure 11. Interaction of a single vortex patch with compound vortices constructed of piecewise continuous patches with the following structure ratios of size/vorticity: (a) $R/-1.0:2R/-0.3:5R/-0.1$; (b) the same as (a) with an additional outer segment of $6R/-0.2$; (c) the same structure as (b) but with vorticity ratios of $-1.0:-0.1:+0.1:+0.2$; (d) the same structure as (a) but with vorticity ratios $-1.0:-0.1:0.1$. 
6. Conclusions

We have examined the interaction of pairs of cyclonic vortex patches for a series of configurations that relate to those observed for tropical cyclones by Holland and Lander (1993) and by those in Part I by Lander and Holland (1993). Vortex patches provide a first-order approximation to the observed atmospheric vortices. Their main attributes are that they can be diagnosed at very high resolution and that they are subject only to advective processes.

When a secondary vortex interacts with a primary vortex, such as a tropical cyclone, both will experience a straining deformation, which is resisted by the tendency of each vortex to maintain a circular shape. If the two systems are of similar size and strength, a bifurcation of behaviour occurs. Inside a critical separation distance rapid merger occurs with the formation of a new composite vortex. At larger separations merger never occurs, and the vortices orbit with oscillatory changes of shape.

The bifurcation behaviour results from a nonlinear interaction between the restoring capacity of each vortex and the strain imposed by the shearing potential flow of the other system. The detailed mechanisms are described in Melander et al. (1988). A simplified expression based on the ratio of strain and restoring capacity was derived to provide an approximate indication of the critical separation distance for merger. This expression indicates that tropical cyclones must approach to within 150–300 km of each other before their core circulations can merge.

Normally one vortex will be weaker and/or smaller than the other and will suffer strong shearing deformation to form a distal-shaped spiral band. Such bands develop and move entirely by shearing deformation and advection, indicating that perhaps gravity-mode processes are not essential for the formation of spiral bands in tropical cyclones. The spiral bands also are associated with local wind maxima. Close to the cyclone core, such maxima may be responsible for development of the secondary eye-wall cycle described by Willoughby et al. (1982). We also have shown that shearing deformation of a large system, such as a monsoon trough, by a tropical cyclone can produce the westerly wind jet often associated with developing tropical cyclones.

The motion of the vortex patches contains several interesting features. When unequal vortices interact, the resulting meanders scale with relative size and intensity. This supports the findings of Holland and Lander (1993) that, as the cyclone increases in intensity, the effect that the mesoscale vortex has on its track will diminish. Importantly, the oscillatory motion of the dominant vortex can be maintained long after the other system has become sheared to the point of being impossible to observe in real-world conditions, or in coarse-resolution numerical models. For example, the mesoscale system disappeared after 24-hours integration in the 70 km resolution barotropic model used by Holland and Lander, but the observed oscillation continued for several days. These results also indicate that a symmetric mesoscale convective system, such as observed by Holland and Lander, is not the necessary condition to produce tropical-cyclone track perturbations. It is quite possible for sheared systems which move into the cyclone circulation to then influence its track.

A second important finding was that high-frequency rack oscillations may result from interactions between a tropical cyclone and nearby synoptic weather systems. We have shown that when a small, intense vortex wraps up a large, weak system, it develops an elliptical shape that rotates at high frequency. Thus the observed trochoidal oscillation of the centre of tropical cyclones (Holland and Lander 1993) may occur from interactions with surrounding weather systems.

We have examined some of the effects of vortex gradients by investigating the
interaction of compound, piecewise continuous, vortex patches, which have a strong core, similar to a tropical cyclone. During the interaction each vortex patch developed a new compound structure by wrapping up vorticity from its neighbour. As a result, simple concepts such as the mutual propagation of interacting vortices on each other’s relative-vorticity gradient do not apply. Further, the relative motion was sensitive to the structure of the outer vortex. For vortices with no change of gradient, the initial distortion of a weak outer cyclonic vortex structure could cause vortex cores initially located far apart to converge. The cores maintained a quasi-symmetric shape until they approached within the merger bifurcation distance, at which time rapid approach and merger occurred. The introduction of a change in vortex gradient to one of the vortices may result in either relative convergence or divergence of the two vortices. One interesting result was that binary interaction may result in the rapid influx of outer fluid from several hundred kilometres radius into the cyclone core.

We conclude from these results that the interaction of binary tropical cyclones, or of tropical cyclones with nearby synoptic systems, will consist generally of a stable orbit and perhaps shearing deformation. However, under some circumstances, considerable sensitivity to initial conditions can be expected. Further analysis of the details of these interactions is desirable.

This study provides a preliminary analysis of vortex interaction to complement the observational study in part I by Lander and Holland (1993) and by Holland and Lander (1992). Part III (Holland and Dietachmayer 1993) extends the analysis to continuous barotropic vortices in an earth-vorticity gradient and provides a discussion of the findings from the complete study.

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