1. Surface Layer Scaling

Recall when we spoke about K-Theory when discussing First Order Closure:

We can define a ***mixing length***, l by $l^2 = c\overline{z'^2}$.

In the surface layer eddies are limited by the earth's surface. It is assumed that $l^2 = k^2 z^2$ where k is the von Karman constant, so:

$$K_E = K_H = K_m = l^2 \left| \frac{\partial \overline{U}}{\partial z} \right| = k^2 z^2 \left| \frac{\partial \overline{U}}{\partial z} \right| \tag{1}$$

In reality, the origin of z for a rough surface because the protrusion of roughness elements above the substrate surface displaces the entire flow upwards. We define the displaced height $z = Z - d$, where d is the zero-displacement height and Z is the height above the substrate surface (height above the actual ground surface).

Hence, for *neutral conditions* with *no buoyancy*, in the *surface layer* (assuming that the stress remains constant throughout the surface layer) we recall the friction velocity, choosing the x-axis appropriately, reduces to:

$$u_*^2 = -\overline{u'w'} = K_m \frac{\partial \overline{U}}{\partial z} = k^2 z^2 \left(\frac{\partial \overline{U}}{\partial z} \right)^2 \tag{2}$$

$$u_* = kz \left(\frac{\partial \overline{U}}{\partial z} \right) \tag{3}$$

Integration gives the famous log-wind profile for neutral conditions:

$$\frac{k\overline{U}}{u_*} = ln(z) + cnt \tag{4}$$

We define the aerodynamic roughness length as z_0 where $\overline{U}(z_0) = 0$. Then,

$$\frac{k\overline{U}}{u_*} = ln\left(\frac{z}{z_0} \right) \tag{5}$$

2. Surface Roughness

The aerodynamic roughness length z_0 is an important parameter, as is the related quantity C_D drag coefficient, because they are used to calculate wind profile, surface stresses and fluxes. Wind profiles require: the zero-plane displacement and the aerodynamic roughness length.

2a. *Zero-Plane Displacement*

The profile relations we derived involve the height $z = Z - d$ measured relative to a reference level termed the zero-plane displacement height d. In this relationship d is chosen so that the wind measured at an actual height Z ($U(Z)$) satisfies:

$$\frac{kU(Z)}{u_*} = ln\frac{Z-d}{z_0} = ln\frac{z}{z_0} \tag{6}$$

The values of d for specific surfaces were determined from observations of neutral wind profiles close to the surface. However, this can be applied to non-neutral surfaces and d is assumed to be independent of stability. We also use the concept of zero-plane displacement height for the profiles of θ and q. In many cases the relationship:

$$\frac{d}{h_c} \approx 2/3 \tag{7}$$

For h_c = height of canopy, gives adequate measure of d, although there is no physical reason for this ratio to be a constant - as d should also be related to the density of the canopy. When $Z > 10h_c$, then d can be ignored

Table 1: Garrat table A6

2b. *Aerodynamic roughness length*

The aerodynamic roughness length z_0 is a surface length scale defined specifically by the log wind law for neutral conditions. Within this definition, the extrapolated wind speed equals zero at the height $z = z_0$.

For aerodynamically smooth flow - that is, when the viscous sublayer is deeper than surface roughness protuberances, experiment shows that:

$$z_0 \approx 0.11\nu/u_* \tag{8}$$

With typical values around 0.01 mm. However, for aerodynamically rough flow, z_0 is a complicated function of the surface geometry. Estimates are done using neutral wind conditions.

- Without many roughness elements, z_0 depends on underlying surface.
- At intermediate element density, drag will increase and so will z_0
- At some point the air will cease to enter the inter-element spaces, and further increases in density will decrease drag and z_0

2c. *Scalar Roughness Lengths*

We defined the surface-layer temperature and humidity profiles with scalar roughness lengths z_T and z_q replacing z_0 in the wind profile relation. Surface temperature and surface humidity are then defined for these heights. The differences arise because heat and water vapor must be transferred by molecular diffusion across the interfacial sublayer. Consequently the *resistance to transfer* momentum between the surface and some height must be less than the resistance to transfer heat or water vapor.

Researchers have defined relationships between z_T, z_q and z_0. These relationships are different for different types of surfaces, whether they are a) *smooth surfaces*, b) *surface with roughness obstacles that are impermeable to the wind flow* or c) *surfaces with permeable or randomly distributed elements*. See Garrat Chapter 4.2 for a thorough description.

3. Monin-Obukhov Similarity Theory

We can take into account the influence of buoyancy through the Richardson number R_f or the Obukhov Length L. The way this is generally done is by taking the dimensionless gradients we had expressed before, which are equal to 1 in neutral conditions, and expressing them as functions of ζ for non-neutral conditions:

290 *Appendices*

The aerodynamic roughness length and the zero-plane displacement

Values of z_0 and d/h_c are given in Table A6 for a range of natural surfaces and values of h_c. Additional values of z_0 and d/h_c, usually in tabular form, can be found in e.g. Sutton (1953), Brutsaert (1982), Pielke (1984) and Stull (1988) though many values are based on the same original source. The wind dependence for flexible crops and grasses is not included here; that for the sea can be deduced from Eqs. 4.5 and 4.23.

Table A6. *Values of aerodynamic roughness length and zero-plane displacement for a range of natural surfaces*

Surface	Reference	h_c (m)	z_0 (m)	d/h_c
soils			0.001–0.01	
grass				
thick	Sutton (1953)	0.1	0.023	
thin	Sutton (1953)	0.5	0.05	
sparse	Clarke *et al.* (1971)	0.025	0.0012	
	Deacon (1953)	0.015	0.002	
		0.45	0.018	
		0.65	0.039	
crops				
wheat stubble	Izumi (1971)	0.18	0.025	
wheat	Garratt (1977b)	0.25	0.005	
		0.4	0.015	
		1.0	0.05	
corn	Kung (1961)	0.8	0.064	
beans	Thom (1971)	1.18	0.077	
vines	Hicks (1973)	0.9	0.023[a]	
		1.4	0.12[b]	
vegetation	Fichtl and McVehil (1970)	1–2	0.2	
woodland				
trees	Fichtl and McVehil (1970)	10–15	0.4	
savannah	Garratt (1980)	8	0.4	0.6
		9.5	0.9	0.75
forests				
pine	Hicks *et al.* (1975)	12.4	0.32	
pine	Thom *et al.* (1975)	13.3	0.55	
		15.8	0.92	
coniferous	Jarvis *et al.* (1976)	10.4[c]–27.5	0.28–3.9	0.61–0.92
tropical	Thomson and Pinker (1975)	32	4.8	
tropical	Shuttleworth (1989)	35	2.2	0.85

[a]Flow parallel to rows.
[b]Flow normal to rows.
[c]Range in h_c for 11 sites; the mean z_0/h_c is 0.076 and the mean d/h_c is 0.78.

Figure 1: from Garrat

Based on this term, we define a dimensionless wind shear:

$$\phi_m(\zeta) = \frac{kz}{u_*}\frac{\partial \overline{U_i}}{\partial z} \tag{9}$$

$$\phi_H(\zeta) = \frac{kz}{\theta_*^{SL}} \frac{\partial \overline{\theta}}{\partial z} \tag{10}$$

$$\phi_E(\zeta) = \frac{kz}{q_*^{SL}} \frac{\partial \overline{q}}{\partial z} \tag{11}$$

(NOTE: substitute the variables for their values in the neutral BL and verify that these variables=1 in the neutral BL)
The forms of the ϕ functions have been extensively studied using observations from many experiments. Observations suggest that:

For $-5 < \zeta < 0$

$$\begin{aligned} \phi_m(\zeta) &= (1 - 16\zeta)^{-1/4} && (12) \\ \phi_H(\zeta) &= \phi_E(\zeta) = (1 - 16\zeta)^{-1/2} && (13) \end{aligned}$$

For $0 < \zeta < 1$

$$\phi_m = \phi_H = \phi_E = 1 + 5\zeta \tag{14}$$

3a. *Integral forms of the flux-gradient relations*

i. Wind For the general, non-neutral case, the surface layer wind profile can be obtained by integrating equation 9:

$$\begin{aligned} \frac{\partial \overline{U}}{\partial z} &= \frac{u_* \phi_m}{kz} && (15) \\ \overline{U}(z) &= \frac{u_*}{k} \int_{z_0}^{z} \left(\frac{dz'}{z'} - \frac{dz'}{z'} + \phi_m \frac{dz'}{z'} \right) \\ &= \frac{u_*}{k} \left[ln \frac{z}{z_0} - \int_{z_0}^{z} (1 - \phi_m) \frac{dz'}{z'} \right] \\ &= \frac{u_*}{k} \left[ln \frac{z}{z_0} - \psi_m(\zeta) \right] \end{aligned}$$

for $0 < \zeta <= 1$ Stable

$$\psi_m = -5\zeta \tag{16}$$

for $1 < \zeta$ Stable

$$\psi_m = -5ln(z/z_0) \tag{17}$$

for $\zeta < 0$ ($x = \phi_m^{-1} = (1 - 16\zeta)^{1/4}$) Unstable

$$\psi_m = 2ln\frac{1+x}{2} + ln\frac{1+x^2}{2} - 2tan^{-1}x + \frac{\pi}{2} \tag{18}$$

In this form, the effects of buoyancy can be interpreted as a deviation of the wind speed from the neutral value.

- In unstable conditions $0 < \phi < 1$ and $\psi > 0$
- In stable conditions $\phi > 0$ and $\psi < 0$

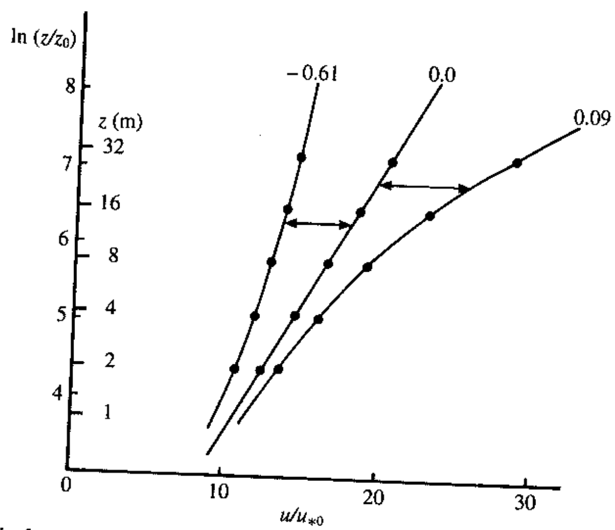

Fig. 3.5 Three wind profiles from the Kansas field data (Izumi, 1971) plotted in normalized form at three values of the gradient *Ri* ($z = 5.66$ m). Both normalized and absolute heights are shown, whilst the magnitude of the horizontal arrows indicates the effect of buoyancy on the wind relative to the neutral profile (see Eq. 3.34).

Figure 2: Garrat

In the general case where we have two wind measurements at heights 1 and 2, we can extend the above expression to:

$$\overline{U_2} - \overline{U_1} = \frac{u_*}{k}\left[ln\frac{z_2}{z_1} - \psi_m(\zeta_2) + \psi_m(\zeta_1)\right] \tag{19}$$

ii. Temperature In analogous form:

$$\frac{k(\overline{\theta} - \theta_0)}{\theta_*^{SL}} = ln\frac{z}{z_T} - \psi_H(\zeta) \tag{20}$$

$$\frac{k(\overline{\theta_v} - \theta_{v0})}{\theta_{v*}^{SL}} = ln\frac{z}{z_T} - \psi_H(\zeta) \tag{21}$$

Here z_T is the surface scaling length for temperature. Formally $\theta = \theta_0$ at $z = z_T$, and z_T is not necessarily equal to z_0. Notice that we are assuming the same nondimensional numbers apply to potential and virtual potential temperature.

for $0 < \zeta <= 1$ Stable

$$\psi_H = \psi_m = -5\zeta \tag{22}$$

for $1 < \zeta$ Stable

$$\psi_H = \psi_m = -5ln(z/z_0) \tag{23}$$

for $\zeta < 0$ ($y = \phi_H^{-1} = (1 - 16\zeta)^{1/2}$) Unstable

$$\psi_H = 2ln\frac{1+y}{2} \tag{24}$$

In the general case where we have two temperature measurements at heights 1 and 2, we can extend the above expression to:

$$\frac{k(\overline{\theta_2} - \overline{\theta_1})}{\theta_*^{SL}} = ln\frac{z_2}{z_1} - \psi_H(\zeta_2) + \psi_H(\zeta_1) \tag{25}$$

$$\frac{k(\overline{\theta_{v2}} - \overline{\theta_{v1}})}{\theta_*^{SL}} = ln\frac{z_2}{z_1} - \psi_H(\zeta_2) + \psi_H(\zeta_1) \tag{26}$$

iii. Humidity In analogous form:

$$\frac{k(\overline{q} - q_0)}{q_*^{SL}} = ln\frac{z}{z_q} - \psi_E(\zeta) \tag{27}$$

for $0 < \zeta <= 1$ Stable

$$\psi_E = -5\zeta \tag{28}$$

for $1 < \zeta$ Stable

$$\psi_E = -5ln(z/z_0) \tag{29}$$

for $\zeta < 0$ ($y = \phi_E^{-1} = (1-16\zeta)^{1/2}$) Unstable

$$\psi_E = 2ln\frac{1+y}{2} \tag{30}$$

In the general case where we have two humidity measurements at heights 1 and 2, we can extend the above expression to:

$$\frac{k(\overline{q_2} - \overline{q_1})}{q_*^{SL}} = ln\frac{z_2}{z_1} - \psi_E(\zeta_2) + \psi_E(\zeta_1) \tag{31}$$

Observations and theory suggest that $\Phi_E = \Phi_H$ and $\psi_E = \psi_H$ and $z_q = z_T$

3b. *Calculating Fluxes using the Flux Profile Method*

As we have shown before, if the stability and the flux or stress is known in advance, then the flux profile method can be used to solve directly for the wind speed or the potential temperature at any height. However, often these relationships are used in reverse, to estimate the flux knowing the mean wind or temperature profile. This is much more difficult. Fore example, u_* appears in a number of places, explicitly and hidden in L, and L is a function of heat flux, which must be estimated from the temperature profile. Solving these equations involves an iterative approach.

Notice how we can use the above expressions to calculate the fluxes as:

$$u_* = \frac{k[\overline{U_2} - \overline{U_1}]}{\left[ln\frac{z_2}{z_1} - \psi_m(\zeta_2) + \psi_m(\zeta_1)\right]} \tag{32}$$

if z_1 = z_0 then U_1=0

$$\overline{(w'\theta')_s} = \frac{-u_* k(\overline{\theta_2} - \overline{\theta_1})}{ln\frac{z_2}{z_1} - \psi_H(\zeta_2) + \psi_H(\zeta_1)} \tag{33}$$

$$\overline{(w'\theta_v')_s} = \frac{-u_* k(\overline{\theta_{v2}} - \overline{\theta_{v1}})}{ln\frac{z_2}{z_1} - \psi_H(\zeta_2) + \psi_H(\zeta_1)} \tag{34}$$

$$\overline{(w'q')_s} = \frac{-u_* k(\overline{q_2} - \overline{q_1})}{ln\frac{z_2}{z_1} - \psi_E(\zeta_2) + \psi_E(\zeta_1)} \tag{35}$$

Given the mean wind (U), pressure (P), humidity (q) and temperature (T) at a level z and at the ”surface”.

1. From the information given calculate the density of air ρ the latent heat L_v, θ and θ_v
2. Calculate u_*, assuming neutral conditions.
3. Calculate $\overline{w'q'}$
4. Calculate $\overline{w'\theta'}$, $\overline{w'\theta'_v}$
5. Calculate L
6. Begin iteration i
   (a) If $L^i > 0$ conditions are stable - calculate $\psi_E = \psi_H$ and ψ_m
   (b) If $L^i < 0$ conditions are unstable - calculate $\psi_E = \psi_H$ and ψ_m
   (c) If $L^i = 0$ conditions are neutral $\psi_E = \psi_H = \psi_m = 0$
   (d) Re-calculate u_*, $\overline{w'q'}$, $\overline{w'\theta'}$ and L using the relationships that depend on stability ζ
   (e) Calculate the difference in the fluxes $\overline{w'q'}$, $\overline{w'\theta'}$ between this iteration and the previous iteration. If the difference is large, continue to iterate until your answers converge.

4. Bulk Transfer Relations

For practical applications, we use drag and bulk transfer coefficients to relate fluxes to mean properties of the flow.

4a. *Drag Coefficient*

Using the relationship 16, and the definition of friction velocity u_*, a drag coefficient $C_D$is defined as:

$$C_D = \frac{(\overline{u'w'}^2)^{1/2}}{\overline{U}^2} = \frac{u_*^2}{\overline{U}^2} = \frac{k^2}{\left[ln\frac{z}{z_0} - \psi_M(\zeta)\right]^2} \qquad (36)$$

$$C_{DN} = \frac{k^2}{\left[ln\frac{z}{z_0}\right]^2} \tag{37}$$

$$\frac{C_D}{C_{DN}} = \left[1 - \frac{\psi_M(\zeta)}{ln\frac{z}{z_0}}\right]^{-2} \tag{38}$$

4b. *Heat Transfer Coefficient*

By analogy with the drag coefficient, a heat transfer coefficient C_H can be defined using the relationship 35 and the definition of $\theta_*^{SL} = -\overline{w'\theta'_s}/u_*$

$$C_H = \frac{Q_H}{\overline{U}(\theta_0 - \overline{\theta})} = \frac{(\overline{\theta' w'}_s)}{\overline{U}(\theta_0 - \overline{\theta})} \tag{39}$$

$$= \frac{(\overline{\theta' w'}_s)}{\frac{-u_*}{k}\left[ln\frac{z}{z_0} - \psi_M(\zeta)\right]\frac{\theta_*^{SL}}{k}\left[ln\frac{z}{z_T} - \psi_H(\zeta)\right]}$$

$$= \frac{k^2}{\left[ln\frac{z}{z_0} - \psi_M(\zeta)\right]\left[ln\frac{z}{z_T} - \psi_H(\zeta)\right]} \tag{40}$$

Where Q_H is the kinematic sensible heat which is the sensible heat divided by ρC_p

$$C_{HN} = \frac{k^2}{\left[ln\frac{z}{z_0}\right]\left[ln\frac{z}{z_T}\right]} \tag{41}$$

$$\frac{C_H}{C_{HN}} = \left[\frac{1}{1 - \frac{\psi_M(\zeta)}{ln\frac{z}{z_0}}}\right]\left[\frac{1}{1 - \frac{\psi_H(\zeta)}{ln\frac{z}{z_T}}}\right] \tag{42}$$

Values of C_{DN}/C_{HN} greater than one indicate the more efficient transfer of momentum than heat as the surface is rougher.

4c. *Moisture Transfer Coefficient*

By analogy with the heat transfer coefficient, a heat transfer coefficient C_E can be defined using the relationship 27 and the definition of $q_*^{SL} = -\overline{w'q'_s}/u_*$

$$C_E = \frac{R}{\overline{U}(q_0 - \overline{q})} = \frac{(\overline{q'w'}_s)}{\overline{U}(q_0 - \overline{q})} \tag{43}$$

$$= \frac{(\overline{q'w'}_s)}{\frac{-u_*}{k}\left[ln\frac{z}{z_0} - \psi_M(\zeta)\right]\frac{q_*^{SL}}{k}\left[ln\frac{z}{z_q} - \psi_E(\zeta)\right]}$$

$$= \frac{k^2}{\left[ln\frac{z}{z_0} - \psi_M(\zeta)\right]\left[ln\frac{z}{z_q} - \psi_E(\zeta)\right]} \tag{44}$$

Where R is the kinematic vertical eddy moisture flux.

$$C_{EN} = \frac{k^2}{\left[ln\frac{z}{z_0}\right]\left[ln\frac{z}{z_q}\right]} \tag{45}$$

$$\frac{C_E}{C_{EN}} = \left[\frac{1}{1 - \frac{\psi_M(\zeta)}{ln\frac{z}{z_0}}}\right]\left[\frac{1}{1 - \frac{\psi_E(\zeta)}{ln\frac{z}{z_q}}}\right] \tag{46}$$

5. Aerodynamic Resistances

The drag, heat and mass transfer coefficients discussed above take into account both turbulent transfer and molecular transfer of a property between the surface and a reference height z in the surface layer. For some applications, it is more convenient to replace the transfer coefficients by ”quasi-resistance” parameters. In this approach, the linking of molecular transfer in the interfacial layer and turbulent transfer in the surface layer is simplified. This relates to the additive property of resistances in series.
By analogy to Ohm’s law (resistance = potential difference / current). For any concentration difference $(\gamma_0 - \gamma)$ and flux F_s

$$r_a = (\gamma_0 - \gamma)/F_s \tag{47}$$

r_a has dimensions of sm^{-1}. The reciprocal r_a^{-1} is the conductance.

5a. *Momentum*

From the definition of C_D (equation 36, we define the *bulk aerodynamic resistance* to the transfer of momentum from a level z to the surface $z = z_0$ as:

$$r_{aM} = \frac{\rho(u(z) - u(z_0))}{\tau_s} = \frac{u(z)}{u_*^2} = (C_D u(z))^{-1} \tag{48}$$

As C_D increases or $u(z)$ increases, the resistance decrease.

5b. *Heat*

From the definition of C_H , we define the *bulk aerodynamic resistance* to the transfer of heat from the surface $z = z_0$ to a level z as:

$$r_{aH} = \frac{\rho C_p(\theta_0 - \theta)}{H_{vo}} = (C_H u(z))^{-1} \tag{49}$$

Where $H_{vo} = \rho C_p \overline{w'\theta_v'}$ is the sensible heat flux.

5c. *Moisture*

From the definition of C_E , we define the *bulk aerodynamic resistance* to the transfer of moisture from the surface $z = z_0$ to a level z as:

$$r_{av} = \frac{\rho(q_0 - q)}{E_0} = (C_E u(z))^{-1} \tag{50}$$

Where $E_o = \rho\overline{w'q'}$ is the latent heat flux.

It is important to note that under near-neutral conditions, the resistance for moisture and heat is higher than for momentum.

Figure 3: Figure 3.8 Garrat

The surface values θ_0 and q_0 must be estimated to use the expressions for the bulk aerodynamic resistance to sensible and latent heat exchange:

$$r_{aH} = \frac{\rho c_p(\theta_0 - \theta)}{H_0} \tag{51}$$

$$r_{aV} = \frac{\rho(q_0 - q)}{E_0} \tag{52}$$