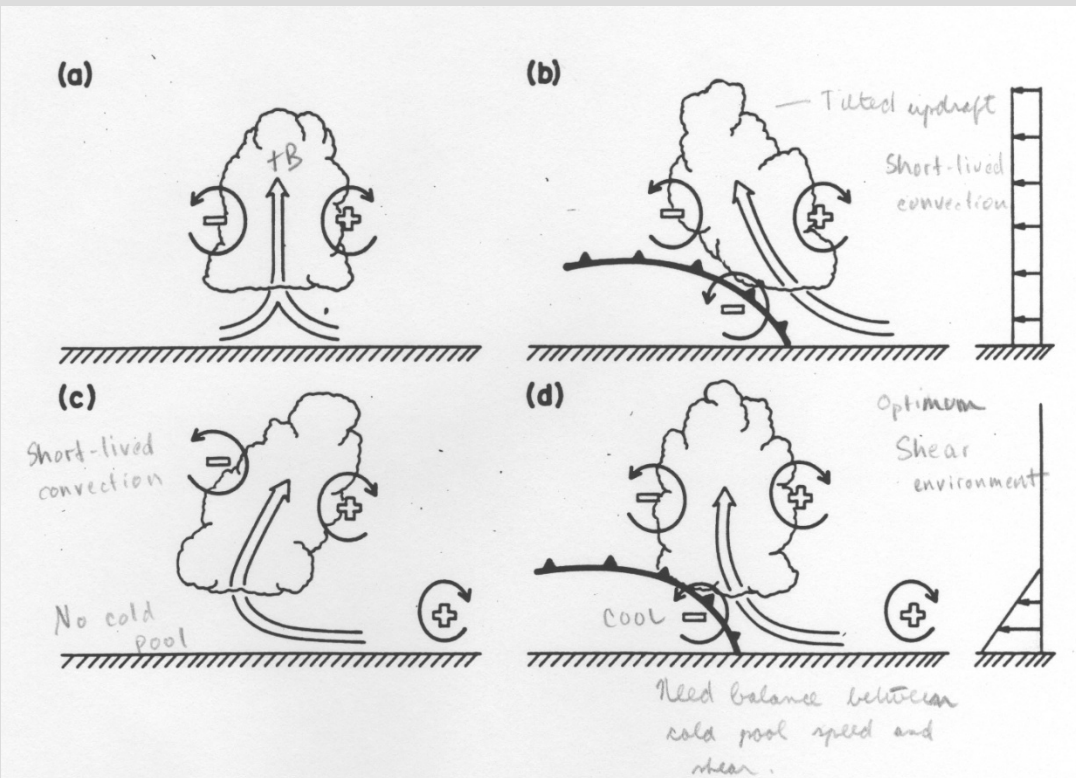
A more detailed and quantitative consideration of organized convection: Part I

RKW Theory

Note: Most lecture notes presented here based on course Daily Weather Laboratory II taught by Prof. Richard Johnson, Colorado State University, Department of Atmospheric Science

Rotunno, Klemp, and Wilhelmson (RKW) Theory: The visual picture



Schematic diagram showing how a buoyant updraft may be influenced by wind shear and/or a cold pool. (a) With no shear and no cold pool, the axis of the updraft produced by the thermally created, symmetric vorticity distribution is vertical. (b) With a cold pool, the distribution is biased by the negative vorticity of the underlying cold pool and causes the updraft to lean upshear. (c) With shear, the distribution is biased toward positive vorticity and this causes the updraft to lean back over the cold pool. (d) With both a cold pool and shear, the two effects may negate each other, and allow an erect updraft. (From Rotunno et al., 1988)

Mesovortices typically develop within mature-to-decaying MCSs. However, so far we have not examined the factors that determine the longevity of MCSs or squall lines. This issue has been studied by Rotunno et al. (1988) [hereafter referred to as RKW], among others. They consider the dynamics of squall lines in terms of the generation of vorticity η about a horizontal axis perpendicular to the squall line by horizontal buoyancy gradients. Referring to the vorticity fields illustrated in Fig. 20, with the x-direction to the right, the vorticity equation is

$$\frac{d\eta}{dt} = -\frac{\partial B}{\partial x} , \tag{7.14}$$

where

$$\eta \equiv \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} ,$$

Vorticity from wind components in x-z plane

and B = total buoyancy = $g\theta_v'/\bar{\theta}_v$. Using mass continuity,

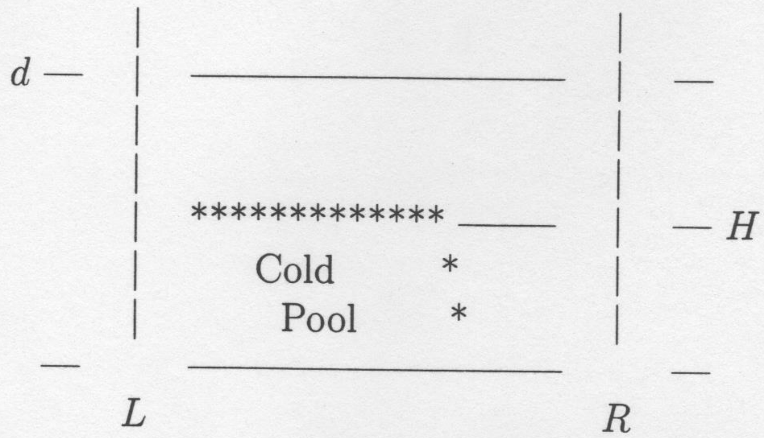
$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 ,$$

(7.14) becomes

$$\frac{\partial \eta}{\partial t} = -\frac{\partial}{\partial x}u\eta - \frac{\partial}{\partial z}w\eta - \frac{\partial B}{\partial x} . \tag{7.15}$$

Expand total derivative

Consider the cold pool illustrated below:



Cold pool on lower left side of the domain

We fix ourselves in a frame of reference moving with the edge of the cold air and integrate (7.15) from a point to the left, $x = L$, to a point to the right, $x = R$, of the cold-air edge, and from the ground to some level, $z = d$, and obtain

$$\underbrace{\frac{\partial}{\partial t}\int_L^R \int_0^d \eta \, dz \, dx}_{\text{tendency}} = \underbrace{\int_0^d (u\eta)_L dz}_{\text{flux at left}} - \underbrace{\int_0^d (u\eta)_R \, dz}_{\text{flux at right}}$$

$$- \underbrace{\int_L^R (w\eta)_d \, dx}_{\text{flux at top}} + \underbrace{\int_0^d (B_L - B_R) dz}_{\text{net generation}} \; . \qquad (7.16)$$

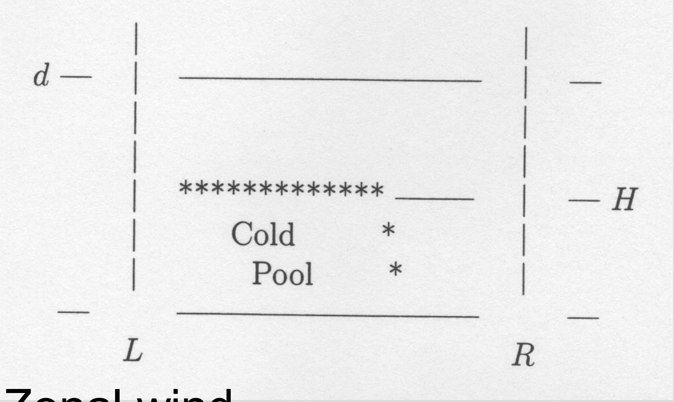
Vorticity tendency = Net advection of vorticity from sides + difference in buoyancy

Simplifying assumptions

Since we are looking for a steady balance, we set the tendency term to zero. Also, in the circumstances investigated by RKW, there is negligible buoyancy of the air approaching the cold pool, so $B_R = 0$. Finally, note that $\eta \approx \partial u/\partial z$ away from the edge of the cold air. Under these conditions, (7.16) becomes

$$0 = \left(\frac{u_{L,d}^2}{2} - \frac{u_{L,0}^2}{2}\right) - \left(\frac{u_{R,d}^2}{2} - \frac{u_{R,0}^2}{2}\right) - \int_L^R (w\eta)_d \, dx + \int_0^d B_L dz \ . \qquad (7.17)$$

Vertical velocity zero at top



Zonal wind zero on lower left

No shear on right side (u=0)

Consider the situation where the cold air is stagnant (relative to the cold-air edge), so $u_{L,0} = 0$, and restricted to a height, $z = H$, where $H < d$. Thus,

$$0 = \frac{u_{L,d}^2}{2} - \left(\frac{u_{R,d}^2}{2} - \frac{u_{R,0}^2}{2}\right) - \int_L^R (w\eta)_d \, dx + \int_0^H B_L \, dz \ . \qquad (7.18)$$

Consider first the case where there is no shear at $x = R$ and a rigid plate at $z = d$. Under these conditions the second and third terms on the right-hand-side of (7.18) vanish and we obtain

$$\begin{aligned} u_{L,d}^2 &= 2\textstyle\int_0^H (-B_L) dz \equiv c^2 \\ &= 2g'H \end{aligned}$$

where $-B_L = -g\Delta\theta/\theta_0 \equiv g'$ and the temperature deficit in the cold pool $\Delta\theta$ has been assumed constant. This reduces to the famous von Kármán formula for speed of gravity current as $d \to \infty$ since $u_{L,\infty} \to u_R$.

Result: zonal wind at the upper boundary travels at the speed of a gravity current due to presence of the cold pool

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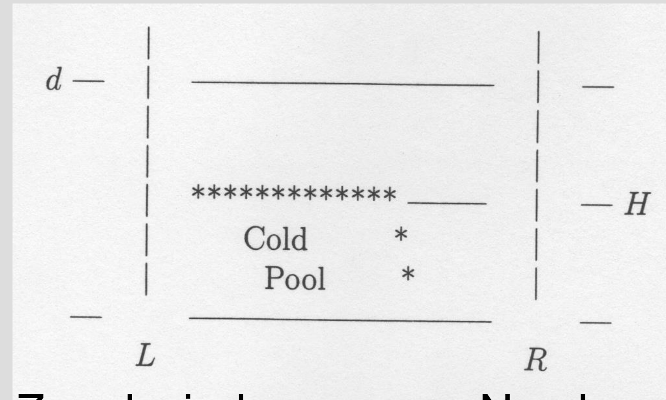
Vertical velocity zero at top



Zonal wind zero on lower left

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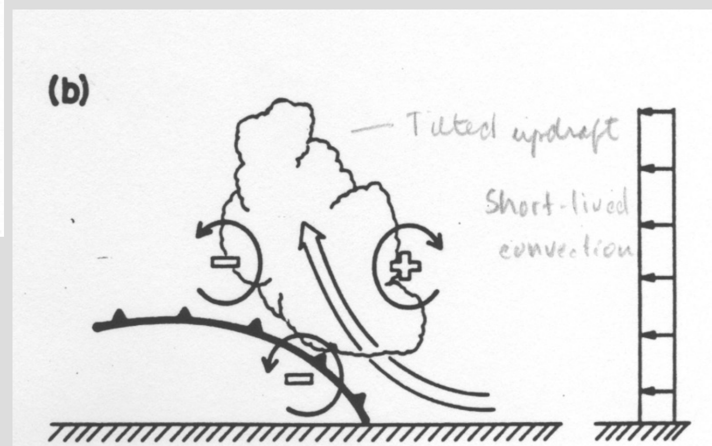


Result: zonal wind at the upper boundary travels at the speed of a gravity current due to presence of the cold pool

Consider now a case with low-level shear (Fig. 19d). Looking for an optimal state where the low-level flow is turned by the cold pool to exit vertically, we have

$$u_{L,d} = u_{R,d} = \int_L^R (w\eta)_d dx = 0$$

so

$$0 = +\frac{u_{R,0}^2}{2} + \int_0^H B_L dz$$

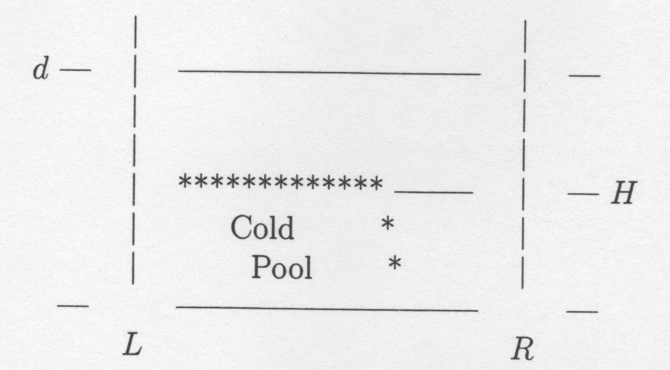
or

$$\Delta u = c\,, \qquad (7.19)$$

where $\Delta u = u_{R,d} - u_{R,0} = -u_{R,0}$ ($u_{R,0}$ is negative).

Equation (7.19) says that the import of positive vorticity associated with the low-level shear just balances the net buoyant generation of negative vorticity by the cold pool in the volume. The RKW hypothesis is then that the *strength and longevity of squall lines are maximized for this balance between the strength of the cold pool and the low-level shear* (a necessary, but not sufficient condition).

Vertical velocity zero at top



Zonal wind zero on lower left

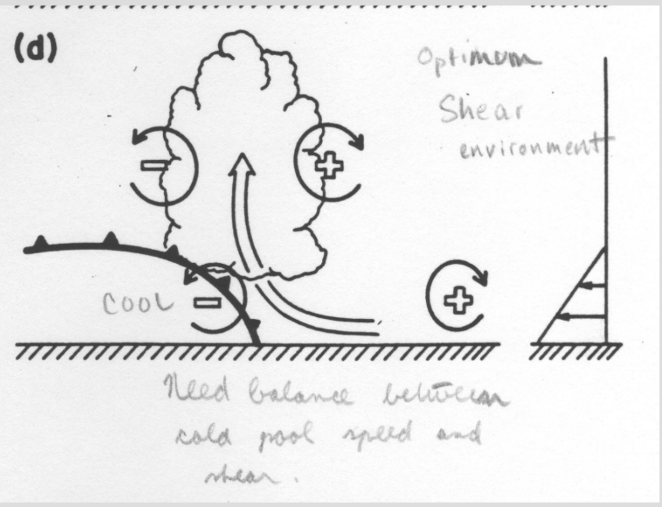
Add zonal shear to this side

OPTIMAL CONVECTIVE STATE =
No inhibition of a vertically upright updraft by cold pool or shear (c = Δu)

c < Δu → Updraft tilts downshear

c > Δu → Updraft tilts upshear

'Low-level' wind shear (Δu) typically means lowest 2-3 km



RKW Theory notes, caveats…

Outflow boundary speed is not necessarily constant, so shear environment may become suboptimal as the squall line matures and outflow boundary speed increases, leading to tilting of updrafts.

BUT…it is the rearward tilting of the updrafts that generates rear inflow and squall line structure! So low-level shear in most severe squall lines generally might be ‘suboptimal’ from perspective of RKW theory.

RKW theory strictly applies to only the tils of the gust front updrafts. But the wind shear profile further aloft becomes more important factor for growth as the storm matures

<u>Bottom line</u>: RKW probably places *too much* emphasis on tilt of gust front updraft in squall line longevity. Most applicable to the issue of convective initiation rather than convective maintenance.

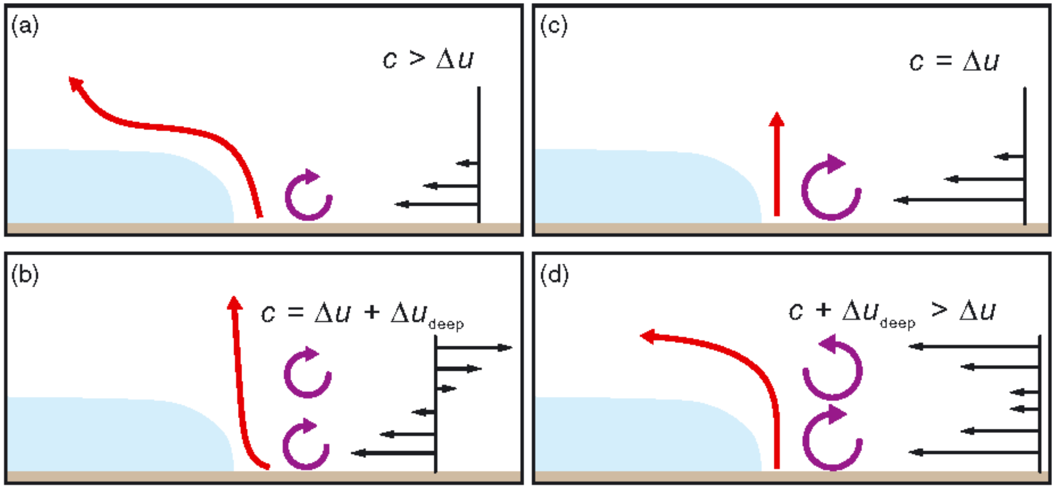


Figure 9.21 Environmental shear over a deep layer (Δu_{deep}) can prevent an updraft from being severely tilted over its own outflow in the event that the low-level shear (Δu) is overwhelmed by the circulation of the cold pool. Rising air parcels are accelerated downshear by the linear dynamic perturbation pressure field; shear present above the cold pool also affects the trajectories of rising air parcels. The effects of the shear also can be understood in terms of vorticity, as illustrated above. (a) The environmental low-level shear is relatively weak (the magnitude of the environmental horizontal vorticity is proportional to the size of the curved arrows) and no environmental shear is present above the cold pool. The horizontal vorticity of the cold pool dominates the horizontal vorticity of the approaching air parcels; therefore, the air parcels are accelerated rearward over the cold pool (the parcel trajectory is indicated with the red arrow). (b) The environmental low-level shear is weak, but significant deep-layer shear is present. The sum of the horizontal vorticity associated with the low-level shear and deep-layer shear is comparable to the horizontal vorticity of the cold pool, resulting in a more vertical updraft. (c) Shear is absent above the cold pool but strong low-level shear and associated horizontal vorticity in the environment balance the horizontal vorticity of the cold pool, leading to an upright updraft; this is RKW theory's optimal state. (d) Strong low-level environmental shear is present and associated with horizontal vorticity that would otherwise be balanced by the horizontal vorticity of the cold pool, but environmental shear above the cold pool is directed rearward over the cold pool. Air parcels are accelerated rearward over the cold pool upon encountering the rearward-directed shear. (Adapted from an image provided by the Cooperative Program for Operational Meteorology, Education, and Training [COMET].)