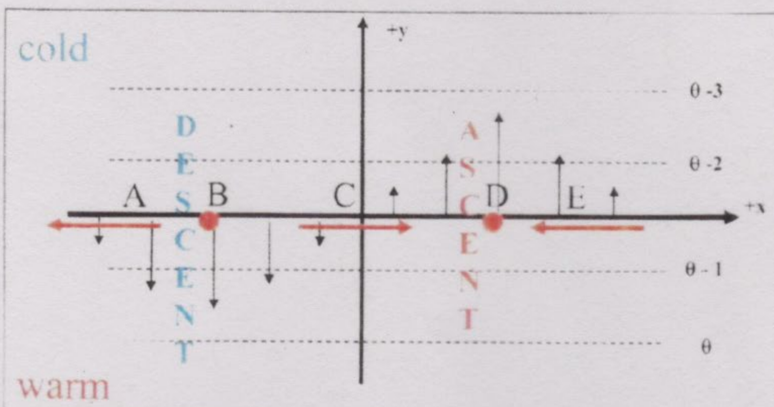
# Computation of Q-vectors (idealized)

- Variation of $v_g$ zonally ($\hat{x}$)
- Variation of $\theta$ meridionally ($\hat{y}$)




$$Q_1 \propto -\frac{\partial u_g}{\partial x}\frac{\partial \theta}{\partial x} - \frac{\partial v_g}{\partial x}\frac{\partial \theta}{\partial y}$$

$$Q_2 \propto -\frac{\partial u_g}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial v_g}{\partial y}\frac{\partial \theta}{\partial y}$$

**Figure 2.10.** Idealized situation illustrating plan view of isentropes (dashed lines), geostrophic winds (solid black arrows), and coordinate axes. Red arrows represent **Q** vectors, and red dots indicate a zero-magnitude **Q** vector. At right, the component **Q**-vector terms are shown, with zero terms denoted by red slash.

Frotogenetic
Geostrophic advection of $\Theta$ strengthens $\nabla\Theta$

Frontolytic
Geostrophic advection of $\Theta$ weakens $\nabla\Theta$

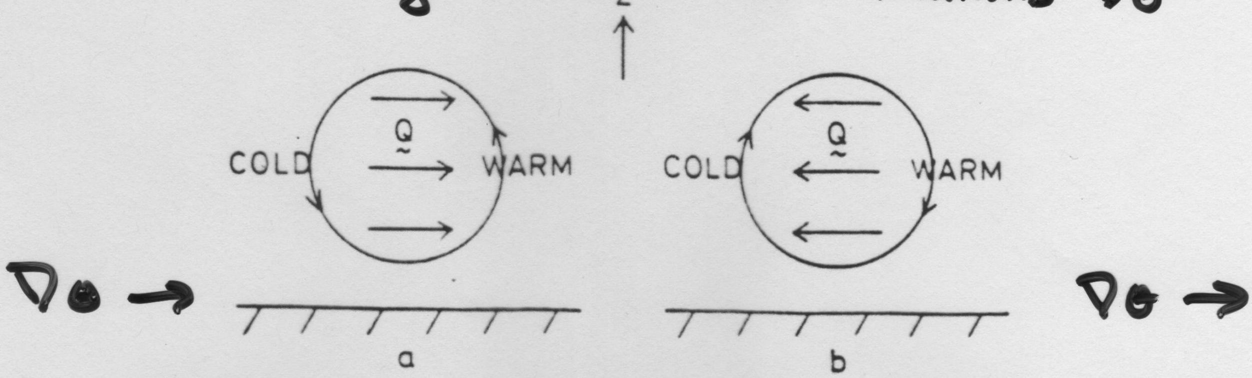



Figure 2. Diagrams showing two-dimensional frontogenetic (a) and frontolytic (b) situations. In (a) the large-scale geostrophic fields are tending to intensify the temperature gradient so that Q is in the direction from cold to warm air. The corresponding cross-frontal circulation is direct with the warm air rising and low-level ageostrophic flow towards the warm side. In (b) the large scale geostrophic fields are tending to weaken the temperature gradient, Q is reversed and so is the direction of the cross-frontal circulation.

B. J. HOSKINS and M. A. PEDDER (1980)

These circulations arise as the atmosphere tries to compensate via ageostrophic motion.

# Mid-latitude cyclone with Isobars and isoallobars

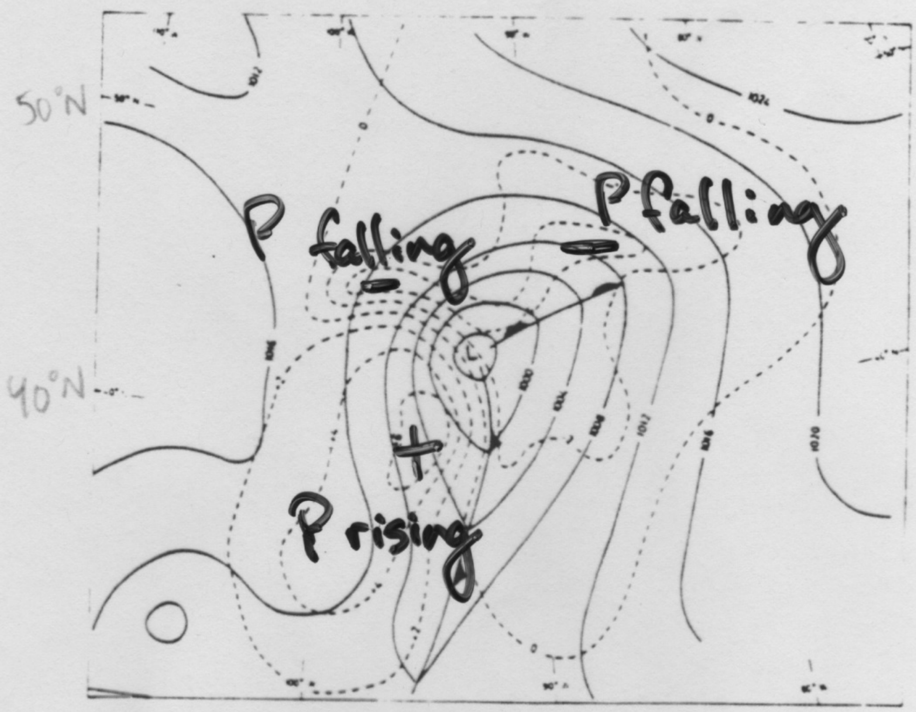



Figure 3. Analysis of surface chart for 0000GMT on 10 November 1975. Isobars are drawn every 4 mb and isallobars every 2 mb $(3h)^{-1}$.

Note: - Isoallobaric wind = $V_{ag}$

- Isoallobars pressure tendency per unit time

# Orientation of Q-vectors in mid-latitude cyclone

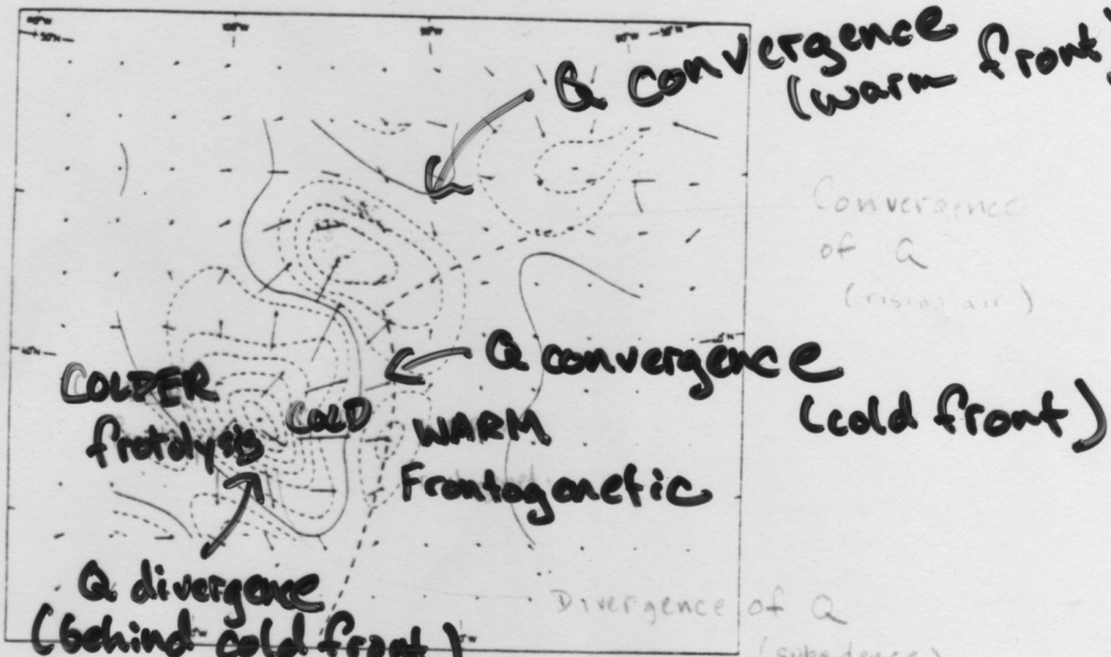



Figure 5. Q and $F_Q = (2g/\theta_0)\,\nabla.Q$ at 700 mb for 0000GMT on 10 November 1975. The length scale for $Q$ is such that the indicated latitudinal distance between grid rows corresponds to approximately $1 \times 10^{-9}$ $K\,m^{-1}\,s^{-1}$. Isopleths of $F_Q$ are every $1 \times 10^{-16}\,m^{-1}\,s^{-3}$ with the zero isopleth being shown as continuous lines. The surface frontal analysis is indicated.

Winds, isentropes

Winds, isobars, frotogenesis

Isobars, Q-vectors, frotogenesis

Vertical motion (ω)

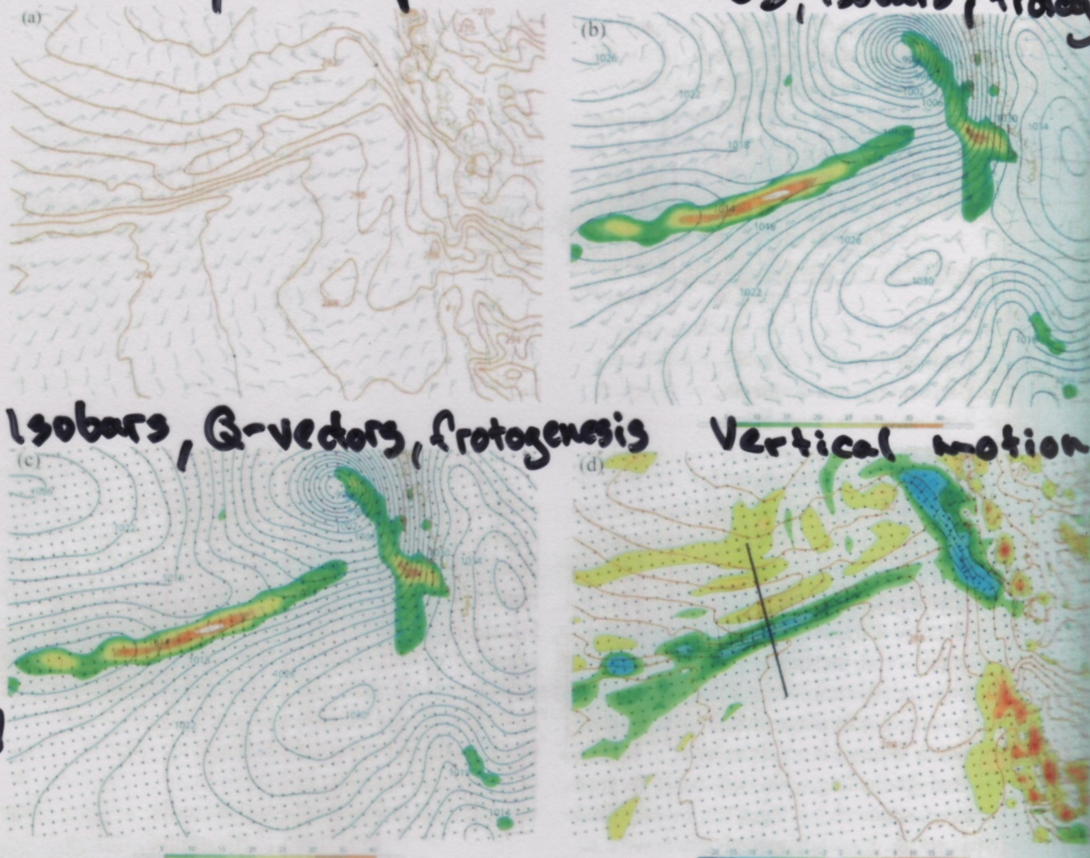
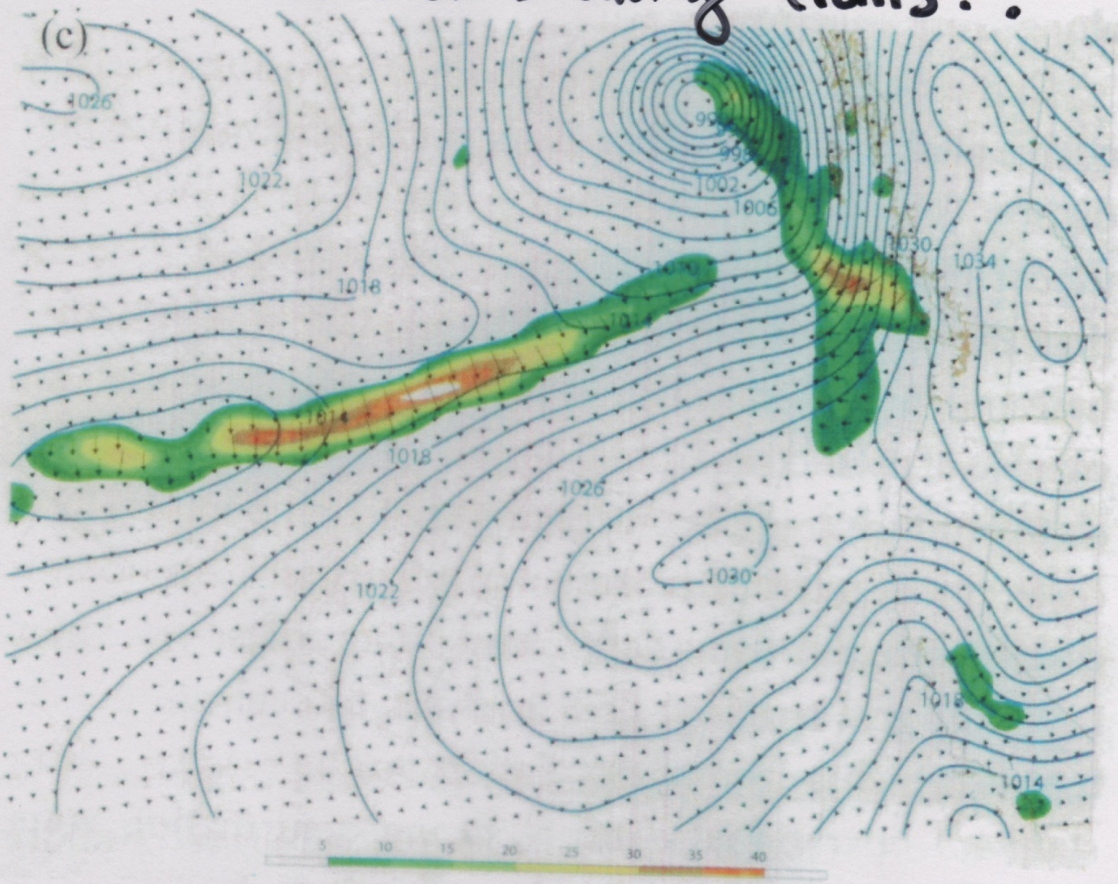
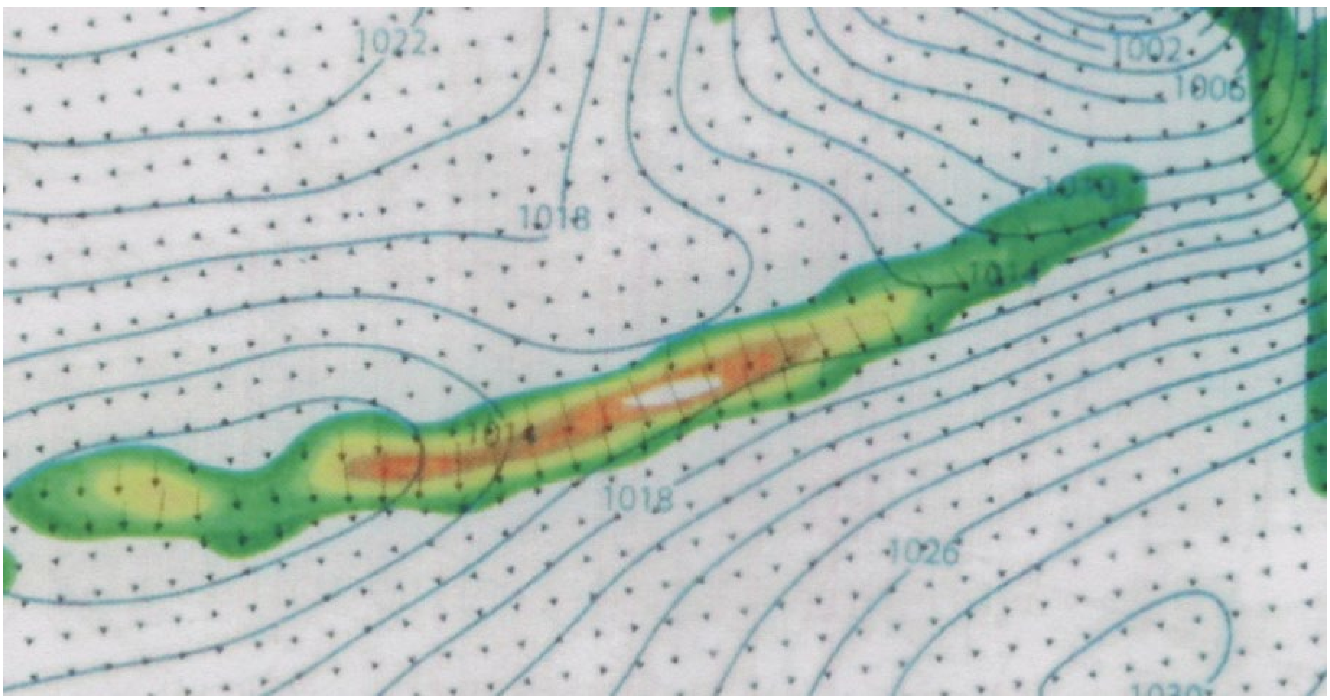



**Figure 6.13.** Diagnosis of North Pacific front with GFS analysis valid 1200 UTC 14 Nov 2008: (a) 10-m wind barbs, 1000-mb isentropes (red, interval is 2 K); (b) 10-*m* wind barbs, sea level pressure, 1000-mb horizontal frontogenesis (shade interval is $5\times10^{-5}$ K $s^{-1}$; (c) as in (b) but 850-mb $Q_n$ (black arrows) replaces 10-m wind; and (d) as in (a), but with 850-mb omega (contour interval is $2\times10^{-2}$ Pa $s^{-1}$, dashed for ascent, solid for descent and shaded as in legend) and 850-mb $Q_n$ replacing 10-m wind.

Equivalent for what plotting for HW #2.

Q-vectors along fronts!!




1022
1002
1006
1018
1014
1014
1018
1026

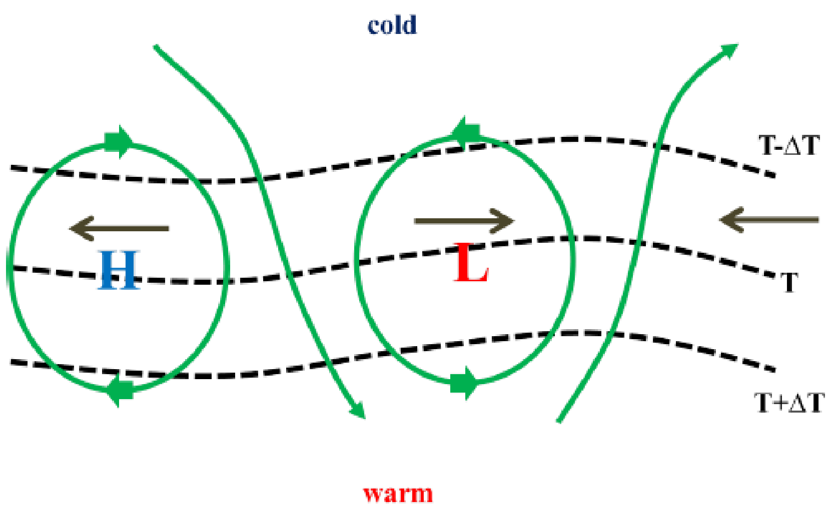



**Figure 2**. **Q** vectors (solid grey arrows) for an idealized trough/ridge pattern. Isotherms are depicted in black dashed lines, with cold air to the north, and streamlines depicting the geostrophic flow are depicted in solid green lines.

*Example 2: Idealized Trough/Ridge Pattern With No Temperature Advection*

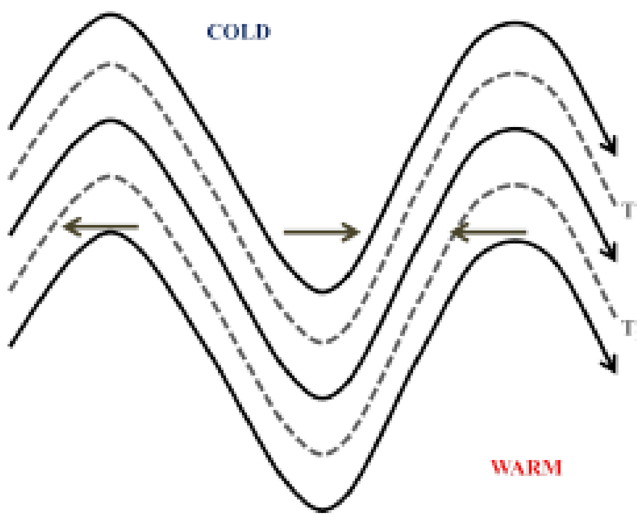



**Figure 4. Q** vectors (solid grey arrows) for an idealized trough/ridge pattern. Isotherms are depicted in grey dashed lines, with cold air to the north, and streamlines depicting the geostrophic flow are depicted in solid black lines.

*Example 3: Confluent Flow in the Entrance Region of a Jet Streak*

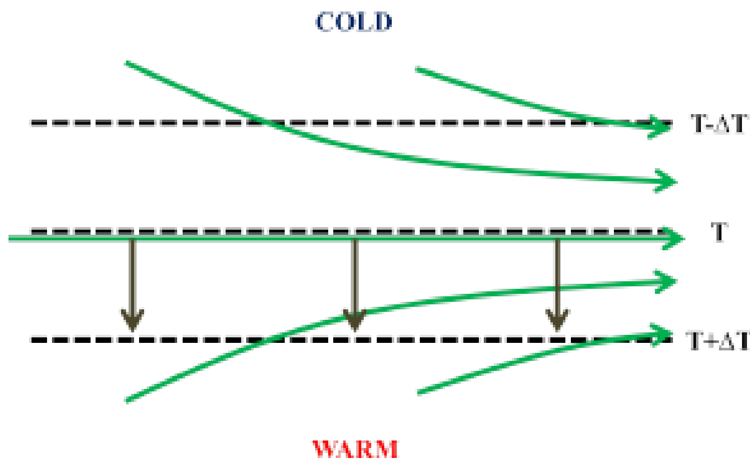
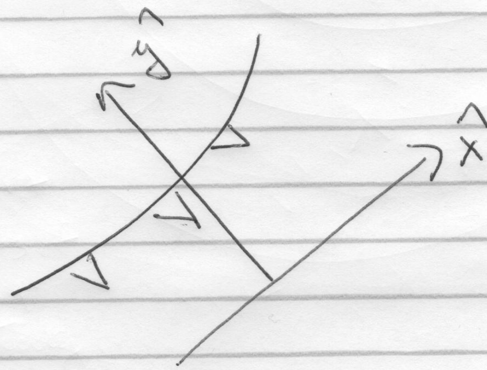



**Figure 5**. Orientation of Q vectors (solid grey arrows) for confluent flow (inferred from the green streamlines obtained from the geostrophic flow) associated with a westerly jet streak. Isotherms are depicted by the dashed black lines with cold air to the north.

Physical interpretation of $Q_1$ and $Q_2$ is that they are contributors to frontogenesis.

If we imagine a coordinate system that is oriented perpindicular to a front.




Frontogenetical equation (F)

$$F = \frac{\partial \theta}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial \theta}{\partial y}\frac{\partial v}{\partial y} + \frac{\partial \theta}{\partial p}\frac{\partial \omega}{\partial y} - \frac{\partial}{\partial y}\left(\frac{\partial \theta}{\partial t}\right)$$

(A) (B) (C) (D)

Synoptic-scale contributions:

(A): Shearing term

(B): Confluence or stretching term
(Most important!)

Mesoscale contributions:

(C): Tilting term

Becomes more important aloft

(D): Diabatic heating term

## Sawyer-Eliassen Equation

In brief → Describes a stream function solution for the cross frontal circulation due to ageostrophic motion.

If we consider Q-vector in the front-relative coordinate system.

$$Q_2 = -\frac{g}{\theta_0}\left[\frac{\partial u_g}{\partial y}\frac{\partial \theta}{\partial x} + \frac{\partial v_g}{\partial y}\frac{\partial \theta}{\partial y}\right]$$

This contains shearing and confluence contributions to frontogenesis equation.

Cross-frontal stream function in y-z plane (normal to front)

$$V_a = -\frac{\partial \psi}{\partial z} \qquad w = \frac{\partial \psi}{\partial y}$$

$$N^2 \frac{\partial^2 \psi}{\partial y^2} + f^2 \frac{\partial^2 \psi}{\partial z^2} = 2 Q_2$$

Sawyer-Eliassen eq. (QG)

→ Simplified Sawyer Eliassen equation for QG theory. Describes an elliptical circulation normal to the front.

## SG Theory (Semi-geostrophic)

To make the Sawyer-Eliassen equation more physically realistic, have to incorporate the advection by the ageostrophic component of the wind.

This is the basis for Semi-geostrophic (SG) Theory, which is going to apply in the vicinity of fronts where QG Theory breaks down.

Original momentum and thermodynamic equations used in Q-vector derivation:

Momentum QG:

$$\frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial x} + v_g \frac{\partial u_g}{\partial y} - f v_{ag} = 0$$

Thermodynamic QG:

$$\frac{\partial \theta}{\partial t} + u_g \frac{\partial \theta}{\partial x} + v_g \frac{\partial \theta}{\partial y} + \frac{\theta_0}{g} N^2 w = 0$$

Now, let's account for the effect of ageostrophic advection!

Momentum SG:

$$\frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial x} + v_g \frac{\partial u_g}{\partial y} - f v_{ag} \boxed{+ v_{ag} \frac{\partial u_g}{\partial y} + w \frac{\partial u_g}{\partial z}} = 0$$

Ageostrophic adv. terms.

Thermodynamic SG:

$$\frac{\partial \theta}{\partial t} + u_g \frac{\partial \theta}{\partial x} + v_g \frac{\partial \theta}{\partial y} + \frac{\theta_0}{g} N^2 w \boxed{+ v_{ag} \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z}} = 0$$

Ageostrophic adv. terms.

From here... the complete derivation of the Sawyer-Eliassen equation for SG theory in shown in Holton, Ch. 9 or in Lackmann text. We wont belabor it here in class notes!

Just let's have the bottom line...

Sawyer - Eliassen equation for SG. theory:

$$\boxed{N_s^2 \frac{\partial^2 \psi}{\partial y^2} + F^2 \frac{\partial^2 \psi}{\partial z^2} + S^2 \frac{\partial^2 \psi}{\partial y \partial z} = 2Q_2}$$

Where ...

$$N_s^2 = N^2 + \frac{g}{\theta_0} \frac{\partial \theta}{\partial z}$$

$$F^2 = f\left(f - \frac{\partial u_g}{\partial y}\right)$$

$$S^2 = -\frac{g}{\theta_0} \frac{\partial \theta}{\partial y}$$

<u>Differences</u> <u>with</u> QG.

- Accounts for horizontal and vertical variation of $\theta$ (i.e. stability) in the vicinity of the front

- Accounts for local variation of absolute vorticity

Does adding the complexities of the effect of the advection by the ageostrophic component of the wind matter??

YES, ABSOLUTELY!!

QG theory breaks down in the vicinity of fronts!!

What SG theory provides

1) More assymetric transverse, frontal ageostrophic circulation, characteristic of frontal tilting
2) Tighter temperature gradient at the front
3) Much shorter time to generate a front. Frontogenesis is hours vs. days with QG.

⟹ ALL characteristics which are more like real atmosphere!!

Assuming only steady geostrophic stretching deformation in a meridional temperature gradient.

From Holton..

$$\frac{D_g}{Dt}\left(\frac{\partial T}{\partial y}\right) = \frac{\partial u_g}{\partial x}\frac{\partial \theta}{\partial y}$$

($\frac{\partial u_g}{\partial x}$ ↑ $K$)

$K$ = deformation term

$K^{-1}$ = e-folding timescale of deformation.

Assuming $u_g$ changs $10\,m\,s^{-1}$ in 1000km...

QG frotogenesis

$K^{-1} \sim 10^5$ s

More than 2 days to increase $\nabla T$ by order of magnitude

SG frontogenesis

Frontogenesis can produce an infinite temperature gradient at surface within less than 12h!!

All due to positive feedback of ageostrophic circulation on frotogenesis...

Example of how confluence term works for frontogenesis.

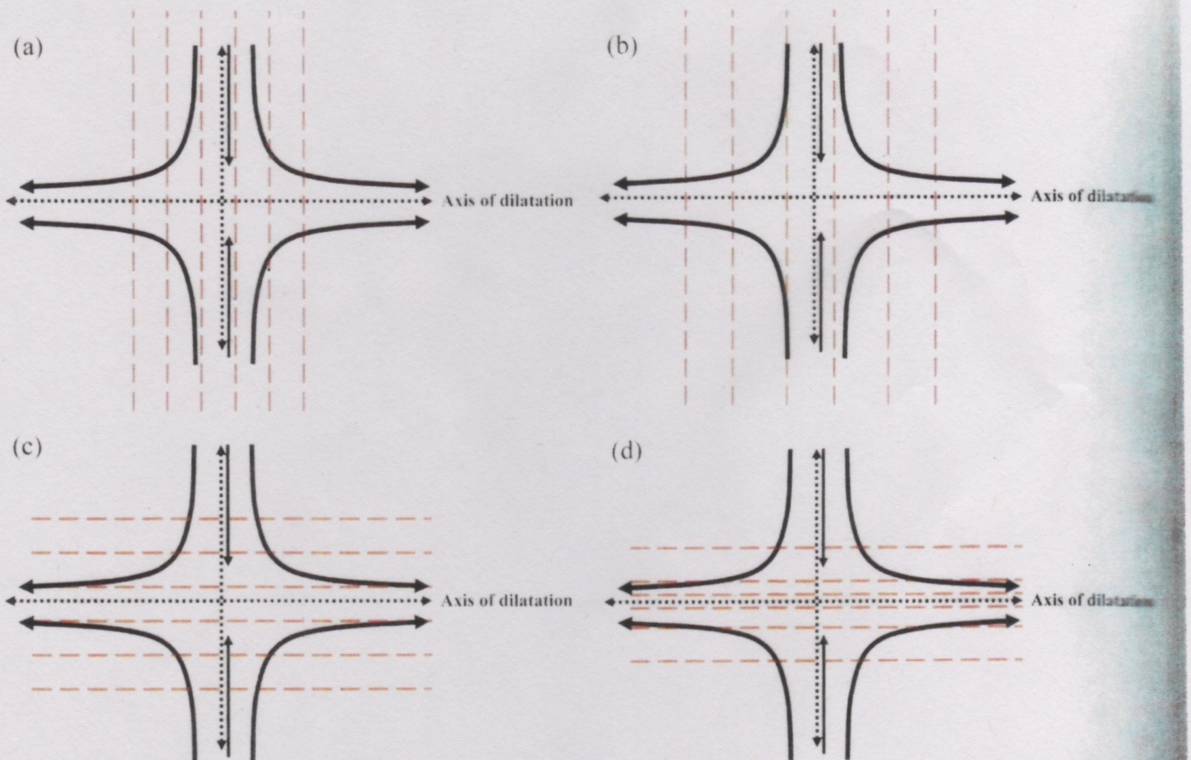
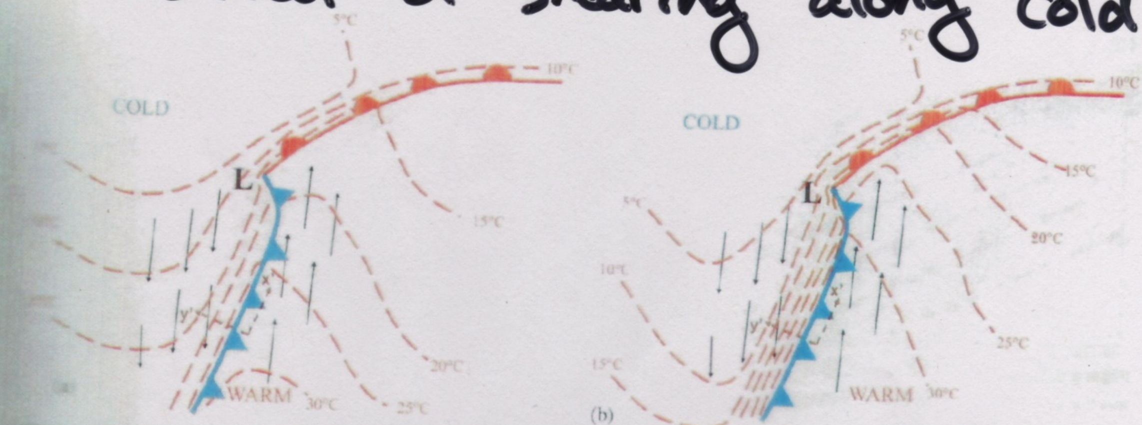
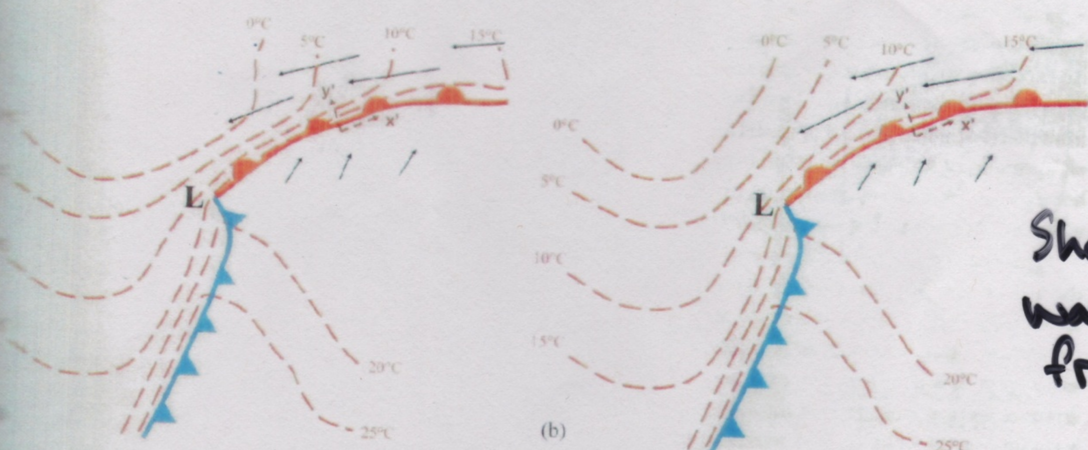



**Figure 6.9.** Idealized schematics of isentrope evolution in a deformation flow. Black lines are streamlines, and red dashed lines are isentropes: (a) initial time, isentropes oriented perpendicular to axis of dilatation; (b) as in (a), but at a later time. (c),(d) As in (a),(b), but with isentropes oriented parallel to the axis of dilatation.

Effect of shearing along cold front

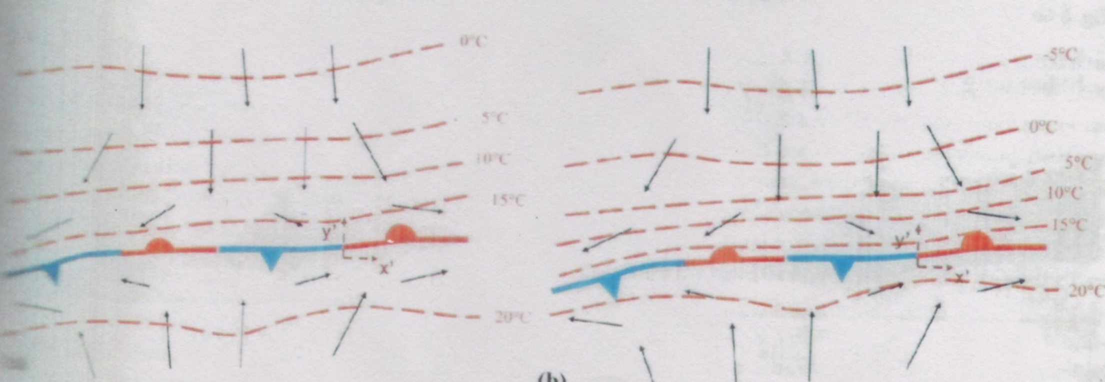



**6.3.** Idealized horizontal plot of near-surface wind vectors (black arrows), isotherms (red dashed contours), and frontal … for (a) initial time, and (b) the same front-relative view at a later time (~24 h later). The rotated front-relative coordinate … shown.




**6.4.** As in Fig. 6.3, but emphasizing the shearing process in the warm-frontal region.

Shearing warm front




**6.5.** As in Fig. 6.3, but demonstrating confluent frontogenesis.

Confluence.

# Q-vector response to confluent frontogenesis

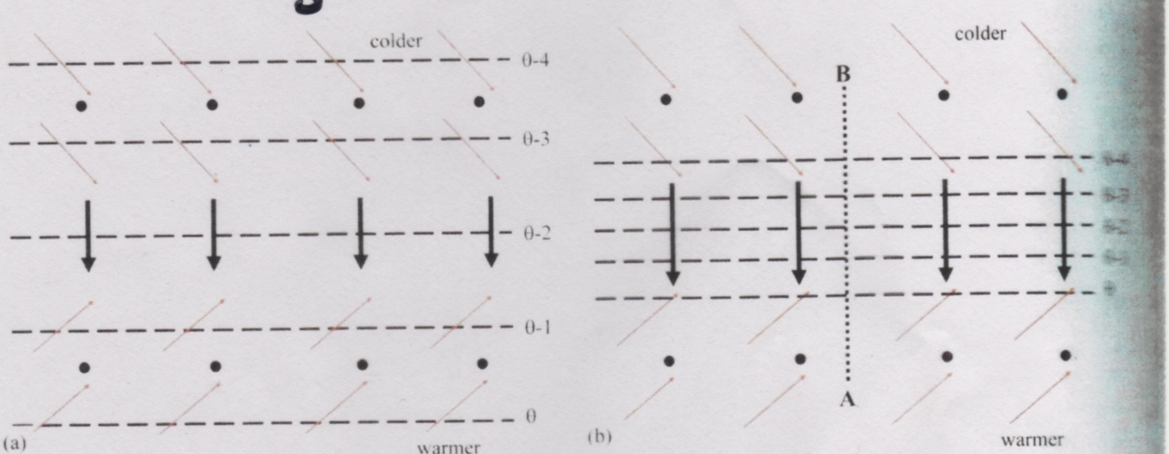



**Figure 6.11.** Schematic representation of confluent frontogenesis with **Q** vectors. Dashed lines represent isentropes, red arrows represent horizontal wind vectors, and thick arrows and dots represent **Q** vectors for (a) initial time and (b) later time. Dotted line A–B in (b) denotes location of cross section shown in Fig. 6.12.

QG theory
Slow intensification of front
~days

SG theory
Fast intensification of front
~hours

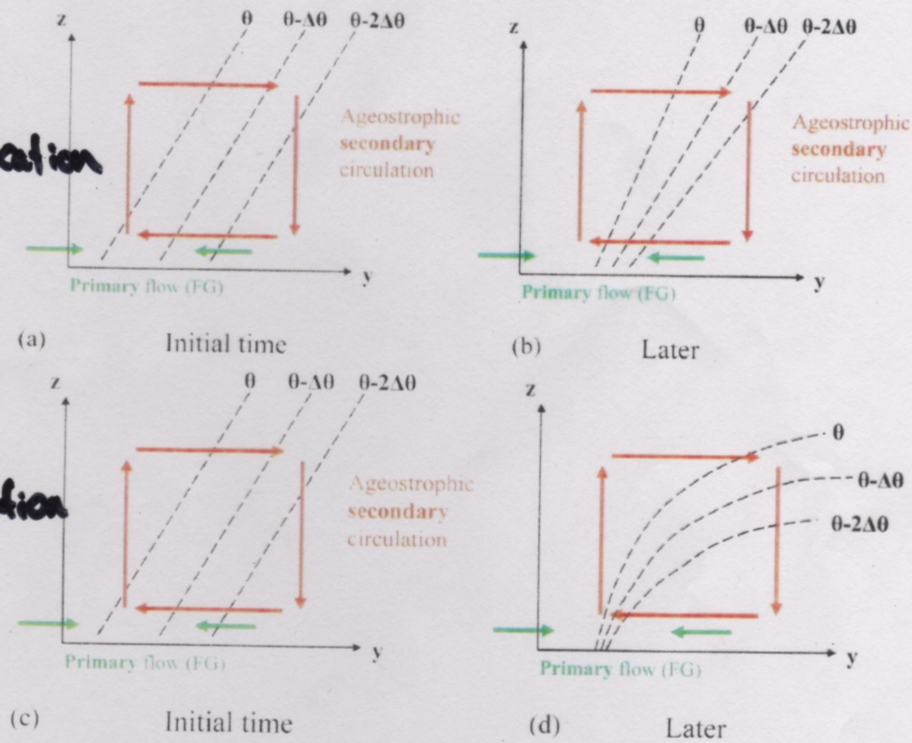



**Figure 6.15.** Idealized comparison of (a),(b) quasigeostrophic frontogenesis with (c),(d) semigeostrophic frontogenesis. Green arrows show primary geostrophic flow, red arrows represent ageostrophic frontal circulation, and dashed black lines are isentropes. Adapted from Bluestein (1986).

Also, greater assymetry in SG case due to the effect of advection by ageostrophic cross-frontal circulation.

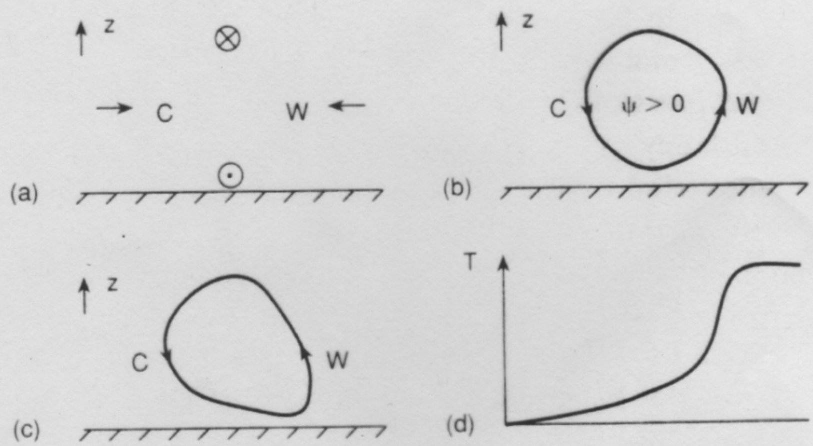
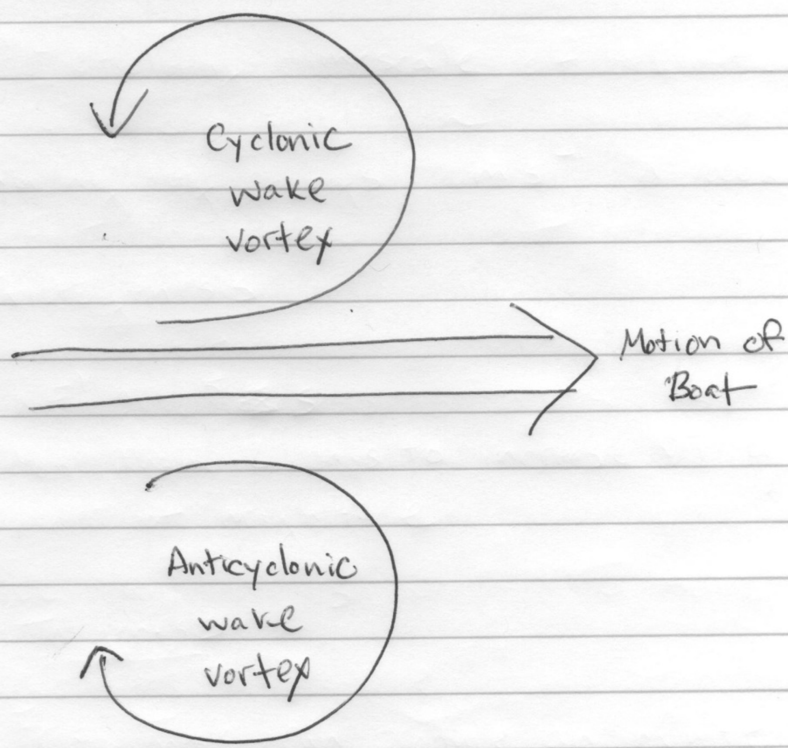



**Fig. 9.5** Role of the ageostrophic circulation in two-dimensional frontogenesis: (a) The basic large-scale flow showing temperature gradient between warm (W) and cold (C) air in thermal wind balance with the along-front component of the geostrophic wind into the section (circle with cross) and out of the section (circle with dot). Geostrophic cross-front flow (shown by arrows) is tending to increase the temperature gradient. (b) The ageostrophic circulation given by quasi-geostrophic theory. (c) The ageostrophic flow given by semigeostrophic theory showing distortion as the absolute vorticity becomes large on the warm side near the surface. (d) The distribution of surface temperature across the front. (Adapted from Hoskins, 1982.)

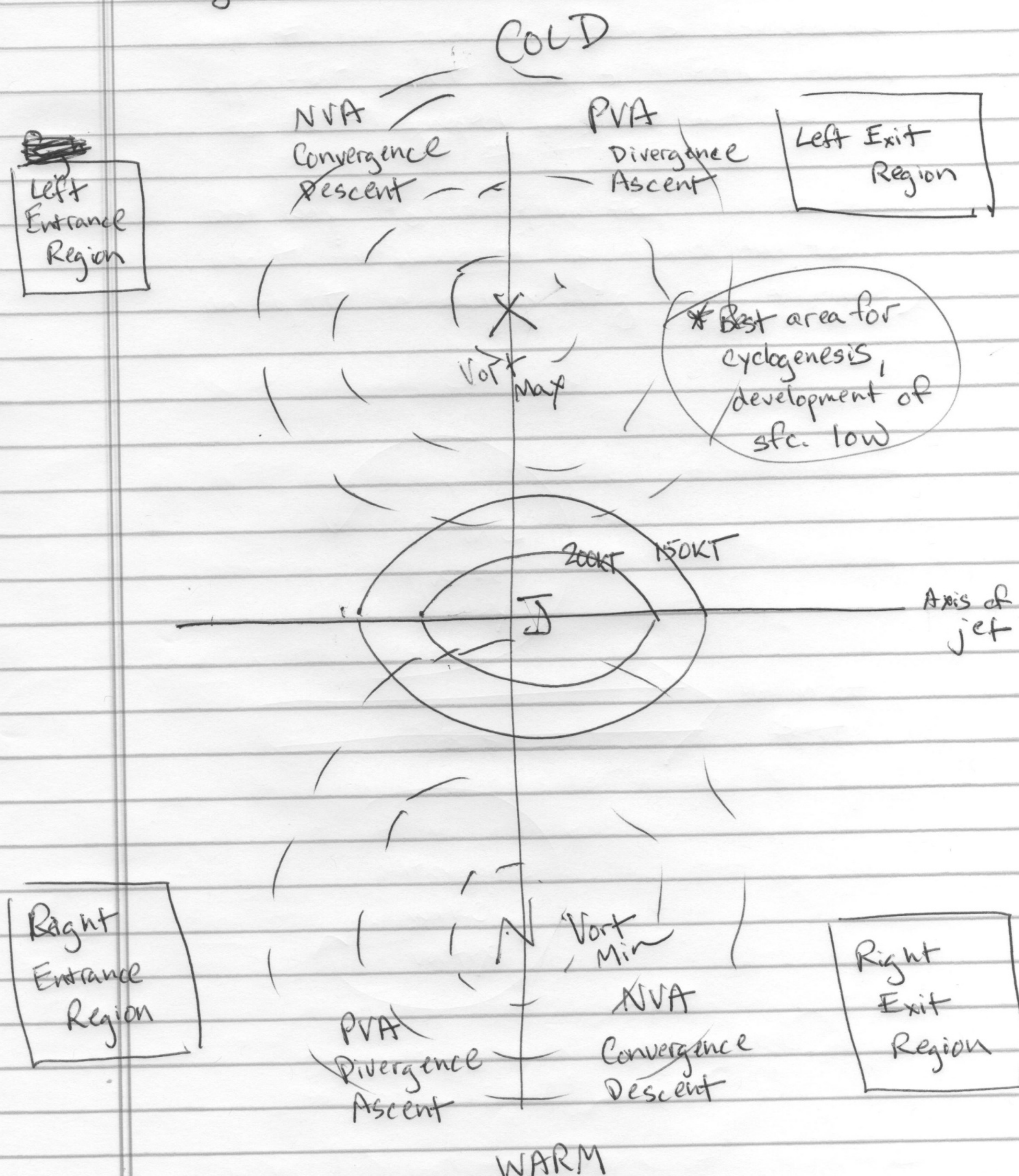
## Jet Streaks

One of the principal features observed on upper-air chart that is directly related to QG-diagnosed vertical motion.

Best way to conceptualize it is to think about the wake of a fluid due to an object in motion (e.g. boat, airplane)

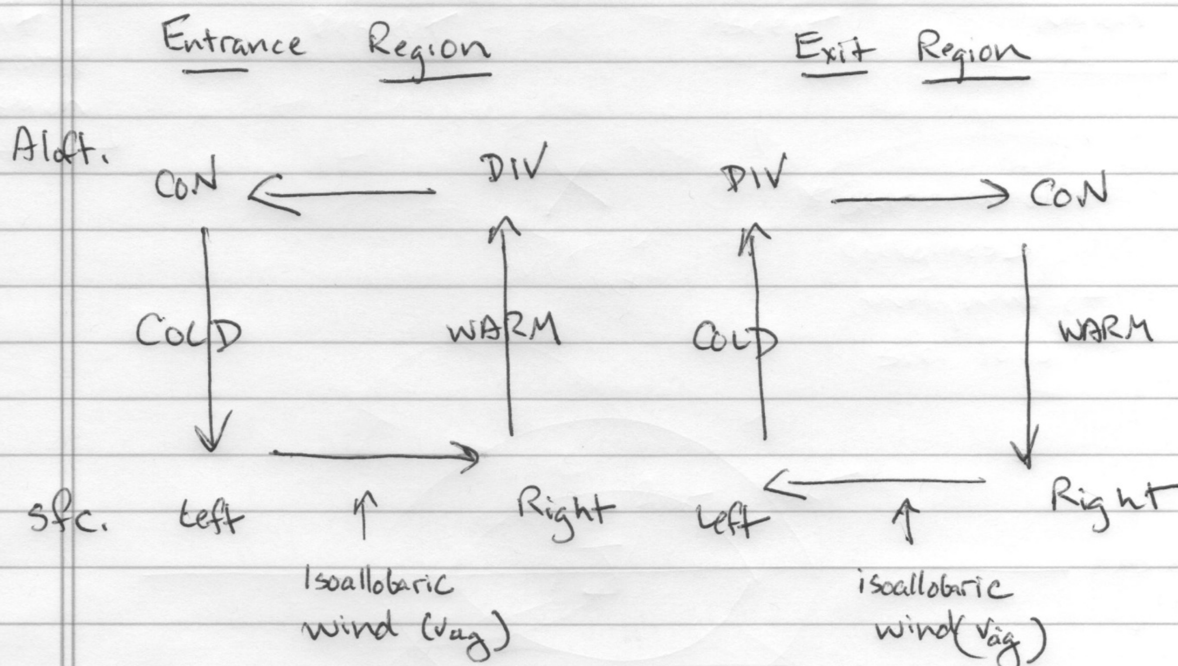


4 QUADRANT JET STEAK MODEL
(e.g. at 200-mb)



# Transverse circulations in Jet Streak

Because there is vertical motion, necessarily implies transverse circulations




| Direct transverse circulation | Indirect transverse circulation |
|---|---|
| - Warm made colder | - Cold made colder |
| - Cold made warme | - Warm made warm |
| - Decreasing $\nabla T$ | - Increasing $\nabla T$ |
| - Frontolysis | - Frontogenesis. |