1. Stream Network

The most common approach to quantitatively describing stream networks was postulated by Strahler (1952).

First Order Streams streams with no tributaries.

Second Order Streams begin at the confluence of two first order streams.

Third Order Streams confluence of two second order streams.

Rule 1 when a stream of a given order receives a tributary of lower order, tis order does not change.

Rule 2 The order of the drainage basin is the order of the stream at the basin outlet.

Some additional definitions:

- Nodes: junctions of streams.
- Links: channel segments between nodes, can be exterior links if they connect to only one node, or interior links within the watershed.
- Magnitude of a Drainage Basin: total number of exterior links it contains.

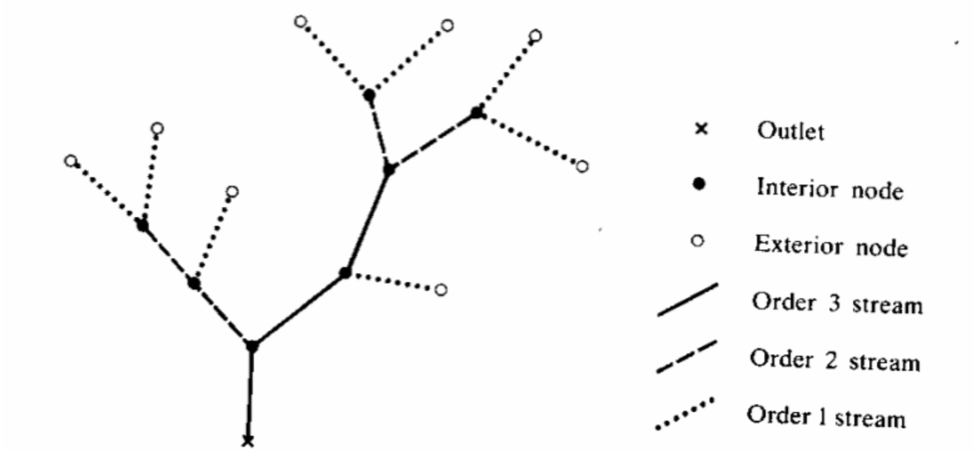
The actual size of the streams designated a particular order depends on the scale of the map or image used. The 1:25,000 is the standard, and measurements at this scale are close to those measured in the field. The interesting consequence of this classification is that the streams show consistent relations we call *laws of drainage network composition*

1a. *Laws of Drainage-Network Composition*

1. Law of Stream Numbers

$$R_B = \frac{N_\omega}{N_{\omega+1}} \tag{1}$$

Range between $3 < R_B < 5$ where R_B is the bifurcation ratio, and N_ω is the number of streams of order ω.



Strahler Stream Ordering Illustrating Nodes and Links

Fourth-Order Channel Network and Basin following Strahler Ordering

Figure 1: Strahler Ordering

2. Law of Stream Lengths

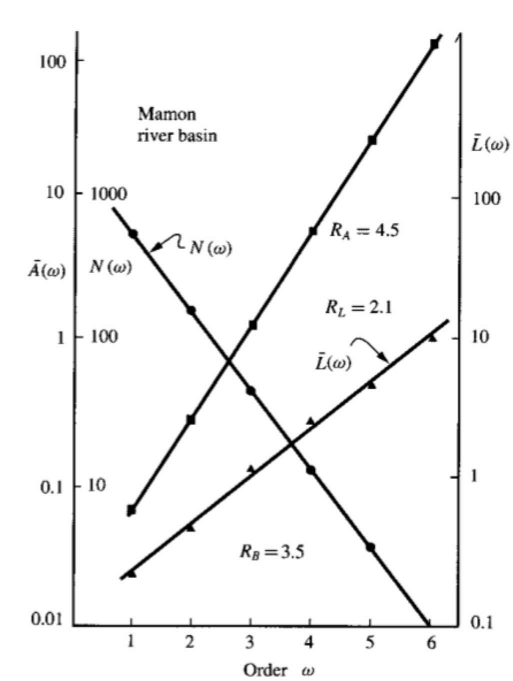
$$R_L = \frac{L_{\omega+1}}{L_\omega} \tag{2}$$

Range between $1.5 < R_L < 3.5$ where R_L is the length ratio, L_w the average length of streams of order ω where $L_\omega = \frac{1}{N_\omega}\sum_{i=1}^{N_\omega} L_{\omega i}$

3. Law of Drainage Areas

$$R_A = \frac{A_{\omega+1}}{A_\omega} \tag{3}$$

Range between $3 < R_A < 6$, where R_A is the drainage-area ratio and A_ω is the average drainage area of streams of order ω where $A_\omega = \frac{1}{N_\omega}\sum_{i=1}^{N_\omega} A_{\omega i}$



Stream Numbers (N_w), Stream Lengths (L_w) and Stream Areas (A_w) versus Order (w) for the Mamon Basin in Venezuela (from Valdes et al. 1979).

Figure 2: Strahler Ordering

The various stream network laws and ratios can be computed for any basin using manual or digital devices, and can be used in a number of ways. For example, the total length of channes of order ω is:

$$L_T = \sum_{i=1}^{N_\omega} L - \omega i = L_1 R_B^{\Omega-\omega} R_L^{\omega-1} \tag{4}$$

where Ω is the order of the highest-order stream. The total length of the stream channels in the entire basin (L_{TB}) is:

$$L_{TB} = \sum_{\omega=1}^{\Omega} \sum_{i=1}^{N_\Omega} L_{\omega i} = L_1 R_B^{\Omega-1} \frac{R_{LB}^{\Omega} - 1}{R_{LB} - 1} \tag{5}$$

Where $R_{LB} = \frac{R_L}{R_B}$

1b. *Stream Network Characteristics*

Drainage density (D_d) of an area A_D is the total length of streams over the area:

$$D_d = \frac{L_{TB}}{A_\Omega} = \frac{L_1 R_B^{\Omega-1}}{A_\Omega} \frac{R_{LB}^{\Omega} - 1}{R_{LB} - 1} \tag{6}$$

Drainage density captures the long-term interaction between climate and basin properties (topography, soils, vegetation.

Stream Frequency Related to the drainage density, the stream frequency F is the number of stream segments per unit area.

$$F = \frac{\sum_{\omega=1}^{\Omega} N_\omega}{A_\Omega} \tag{7}$$

Network Width Function measures the number of links at a given distance x from the basin outlet $W(x)$.

$$W(x) = \sum_x N_x \tag{8}$$

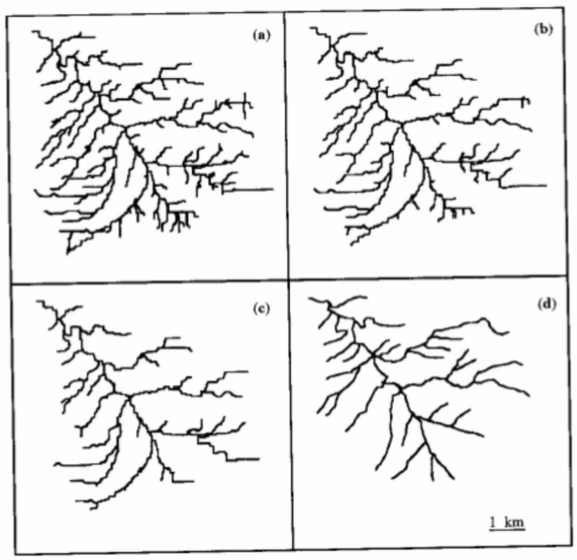


Illustration of Stream Network at Different Drainage Densities

Figure 3: Strahler Ordering

2. Rainfall-Runoff Modeling

Forecasting and predicting flood peaks and runoff volumes due to large rain and snowmelt. Hydrologists use models to simulate the stream response to a water input event (**rainfall-runoff models**. These models should simulate the physical processes including changes due to extreme rainfall, land-use changes, changes in climate.
While we can do this by solving the equations for saturated and unsaturated soil, this is computationally very expensive and requires a lot of data and field observations. Commonly used models are based on conceptual and empirical relations. Rainfall-runoff models are used to generate design floods and floods from actual storms.

A *design flood* is a flood of a specified return period that is used in the design of culverts, bridges, flood-retention basins, levees, dam spillways or floodplain-management plans. The design return period depends on the consequences of exceedence. For example, you would use a small return period (10yr) for a small culvert but a 100yr return value for a large dam, one could also use the probable maximum flood for a very economically costly structure.One of the important questions is the duration of the storm for the design rainfall. The critical duration increases with watershed area, and is on the order of the time of concentration.

We can also use these models to forecast flooding from an in-progress storm. These models are usually more physically based and complex, include snowmelt and infiltration. The models have to be calibrated, and the procedure of "parameter estimation" must be used.

Remember that the estimate of W_{eff} is very important. There are several methods to do this when you don't have detailed observations of storage components (soil moisture).

Figure 4: Figure 9-40

2a. *The Rational Method*

Postulated a proportionality between peak discharge q_{pk} and rainfall intensity i_{eff}.

$$q_{pk} = u_R C_R i_{eff} A_D \tag{9}$$

where u_R is a unit conversion factor, A_D is the drainage area, C_R is the runoff coefficient. This is a simplified model for basins with negligible surface storage like small suburban and urban watersheds. The duration of the rainfall is taken as the time of concentration of the watershed (Table 9.9). The only parameter C_R is the ratio of peak streamflow per unit area to rainfall intensity, and varied from .05 for gently sloping lawns ot 0.95 for highly urbanized roofs or pavements.

3. SCS Curve-Number Method

The most widely used rainfall-runoff model in the US is the SCS method. Makes use of soils information. First, the effective rainfall is estimated and then the peak discharge and the entire runoff hydrograph are calculated.

1. $Weff$ is related to total rainfall W and watershed storage capacity V_{max} for small watersheds via

   $$W_{eff} = \frac{(W - 0.2V_{max})^2}{W + 0.8V_{max}} \tag{10}$$

   This mimics the physical processes much like the Green Ampt infiltration equation. The NRCS (US National Resources Conservation Service) has maps of the soils in the US, and one can determine V_max from these maps along with land cover inforamtion. The soil type is mapped o one of four groups (A,B,C,D). Table 9-12 gives the SCS curve numbers assigned to each hydrologic soil group under various land uses. V_max is determined from these curve numbers CN as:

   $$V_{max} = \frac{1000}{CN} - 10 \tag{11}$$

   where V_{max} is in inches. Note that this is an empirical dimensionally inhomogeneous equation and the constants differ when other units are used. One can adjust the curve numbers to reflect antecedent wetness of the watersheds. However, the curve numbers used in Table 9-12 are used for most design purposes. When a watershed consists of more than one soil/land-use comples, we use a weighted average curve number.

2. The peak discharge q_{pk} is estimated by assuming that the runoff hydrograph is a triangle, with a time of rise,T_r given by:

$$T_r = 0.5T_W + 0.6T_c \tag{12}$$

T_W is the duration of excess rainfall and T_c is the time of concentration given by one of the formulas in Table 9.9. The time base of hydrograph is:

$$T_b = 2.67T_r \tag{13}$$

We calculate the total runoff as the triangular area:

$$Q_{ef} = 0.5q_{pk}T_b \tag{14}$$

where

$$q_{pk} = \frac{484W_{eff}A_D}{T_r} \tag{15}$$

While the SCS method is widely used, it is unwise to accept the method uncritically. Field observations are always advisable

4. Unit Hydrograph

The *unit hydrograph* is the characteristic response of a given watershed to a unit volume of effective water input applied at a constant rate for T_W hours. The basic hypothesis is that the response of the watershed is linear. The unit hydrographs for a watershed can be constructed from observations of input and response for several significant storms of approximately equal duration.

1. Choose four or five hydrographs from intense storms of approximately equal duration.
2. Plot each hydrograph and separate the base flow.
3. Convert the W_{eff} to in/hr or cm/hr
4. Multiply the ordinates by $1/W_{eff}$
5. Plot the hydrographs on the same graph

6. Determine the average of all the peaks and the average of all the times of peak

7. Sketch the composite unit hydrograph.

8. Adjust the area under the curve to be 1 in or cm

Note that if we have to rainfall events occurring one after the other, the response hydrograph will be given by the sum of the ordinates of the two responses. Given a $T_W' - hr$ unit hydrograph, we can obtain the $nTw' - hr$ unit graph for $n = 1, 2, 3..$by adding the n unit graphs, each lagged by Tw' and dividing the resulting ordinates by n.

Unit graphs for duration T_W less than T_W' can be obtained from the construction of the *S-hydrograph*:

1. A series of $T_W' - hr$ unit hydrographs is plotted beginning at successive intervals of T_W' hr.

2. The ordinates of all the unit graphs are added to give the ordinates of the S-hydrograph until the ordinate becomes effectively constant.

3. Plot the S-hydrograph twice lagged by T_W and subtract the ordinates for the lagged curve from those of the first curve.

4. Multiply the ordinates found in Step 3 by T_W' to give tohe ordinates for the $T_W - hr$ unit graph

The S-hydrograph represents the hydrograph of a sotrm of infinite duration at an intensity of 1 unit / T_W'.

4a. *Instantaneous Unit Hydrograph*

If the input duration T_W used to define the unit hydrograph is allowed to become infinitesimal, the resulting response function is the *instantaneous unit hydrograph* the response of the watershed to a unit volume of input applied instantaneously. This permits the use of continuous mathematics in developing the transfer function from measurements of the watershed response hydrograph, $q_ef(t)$ to a continous input $w_{ef}(t)$