1. Water in Soils: Infiltration and Redistribution

Both rainfall and snowfall lead to a soil hydrologic response. Approximately 76% of the world’s land-area precipitation infiltrates and provides water used by plants and almost all groundwater. It is important to know how the surface will react to rain or snowmelt.

**Infiltration** Movement of water from the soil surface into the soil

**Exfiltration** Evaporation from the upper layers of the soil

**Redistribution** Movement of infiltrated water in the unsaturated soil

**Interflow** Flow that moves downslope.

**Percolation** Downward flow in the unsaturated zone.
**Recharge** Movement of percolating water from the unsaturated zone to the saturated zone.

**Capillary Rise** Movement from the saturated zone upward into the unsaturated zone due to surface tension.

**Plant Uptake** Root water uptake for plant consumption and transpiration.

1a. **Material Properties of Soil**

We model quasi-homogeneous as a matrix of soil grains between which are interconnected pore spaces that can hold varying proportions of water and air - we will not take organic composition into account separately. Particle size distribution of soils is characterized by soil texture determined by the proportion of sand, silt and clay.

1. Size of the soil pore spaces is approximately equal to the grain size.

2. The grain size distribution determines the amount of available porosity.

3. Soil texture is typically used to determine grain (or particle) size distribution.

4. Soil texture consists of the weight fractions of silt, clay and sand.

The properties of soil will provide a quantitative description of unsaturated zone processes, including infiltration. For a soil column, we can define:

**Particle Density** $\rho_m$ weighted average density of the mineral grains making up a soil.

\[
\rho_m = \frac{M_m}{V_m} \tag{1}
\]

where $M_m$ is the mass and $V_m$ is the volume of the mineral grains. We usually estimate based on the mineral composition of the soil and assume 2650 kg m$^{-3}$ which is the density of mineral quartz.

**Bulk Density** $\rho_b$ is the dry density of the soil.

\[
\rho_b = \frac{M_m}{V_s} = \frac{M_m}{V_a + V_w + V_m} \tag{2}
\]
Figure 2: default

USDA Soil Texture Triangle
where $V$ is volume and the subscripts refer to soil ($s$), air ($a$), liquid water ($w$) and minerals ($m$). Bulk density increases with depth due to compaction. We measure it as the weight of the soil that has been dried at 105°C divided by original volume.

**Porosity** $\phi$ is the proportion of pore spaces in a volume of soil.

$$\phi = \frac{V_a + V_w}{V_s} = 1 - \frac{\rho_b}{\rho_m}$$  \hspace{1cm} (3)

It decreases with depth because of macropores at the surface and compaction. We measure it by calculating $\rho_b$ and assuming $\rho_m = 2650$. Finer grained soils have higher porosities than coarse soils.

**Volumetric Water Content** $\theta$ ratio of water volume to soil volume.

$$\theta = \frac{V_w}{V_s}$$  \hspace{1cm} (4)

It could range from 0 to $\phi$, but in reality the range is smaller. We measure it by weighting the soil of known volume ($M_{swet}$), oven drying ($M_{sdry}$), re-weighting it and calculating

$$\theta = \frac{M_{swet} - M_{sdry}}{\rho_w V_s}$$  \hspace{1cm} (5)

**Degree of Saturation** ($S$) or wetness is the proportion of pores that contain water:

$$S = \frac{V_w}{V_a + V_w} = \frac{\theta}{\phi}$$  \hspace{1cm} (6)

1b. **Soil Water Flow**

We can begin with the flow $Q$ per unit cross-sectional area $A$ at a pore velocity $v$:

$$Q = \phi v A$$  \hspace{1cm} (7)

where $\phi$ is the soil porosity. If we define the Darcy velocity $q = \phi v$, we can write:

$$q = \frac{Q}{A}$$  \hspace{1cm} (8)

While $q$ is the volumetric flow rate per unit bulk area, the true velocity
for fully saturated conditions and

\[ v = \frac{q}{\phi} \]  

(9)

for unsaturated conditions.

**Darcy’s Law** governs \( q \), the flow and redistribution in unsaturated porous media.

\[
q_x = -K_h \frac{d(z + p/\gamma_w)}{dx}
\]

(11)

\[
= -K_h \left[ \frac{dz}{dx} + \frac{d(p/\gamma_w)}{dx} \right]
\]

(12)

Where \( q_x \) is the volumetric flow rate in the x-direction per unit cross-sectional area, \( z \) is the elevation above arbitrary datum, \( p \) is the water pressure, \( \gamma_w = \rho_w g \) is the weight density of water and \( K_h \) is the hydraulic conductivity. Darcy’s law represents flow at a *representative elemental volume* of the soil. Physically it means that flow occurs in response to spatial gradients of mechanical potential energy (gravitational potential and pressure potential). We will only consider flows in the vertical:

\[
q_z = -K_h \left[ \frac{dz}{dz} + \frac{d(p/\gamma_w)}{dz} \right]
\]

(13)

\[
= -K_h \left[ 1 + \frac{d(p/\gamma_w)}{dz} \right]
\]

(14)

Since \( \gamma_w \) is effectively constant for hydrologic problems that do not involve temperature or salinity, we use the pressure head, defined as \( \psi = \frac{p}{\gamma_w} \) which is expressed in length dimensions. The magnitude of the gravitational potential energy gradient will always equal one (+ or - depending on the direction of flow and definition of the coordinate system, in this case +1 going up and -1 going down). \( p \leq 0 \) for the unsaturated flows considered here. Note that in unsaturated flows, the hydraulic conductivity and the pressure head for a given soil are functions of the soil-water content \( \theta \) so we write Darcy’s Law as:
\[ q_z = -K_h(\theta) \left[ 1 + \frac{d\psi(\theta)}{dz} \right] \] (15)

The way that \( \psi \) and \( K_h \) vary as a function of soil moisture is extremely important for hydrologic flows.

**Pressure** It is conventional to measure pressure relative to atmospheric pressure. \( p > 0 \) and \( \psi > 0 \) in saturated flows and \( p < 0 \) and \( \psi < 0 \) in unsaturated flows. The water table is the surface at which \( p = 0 \). Negative pressure is called tension or suction and \( \psi \) is called the tension head, matric potential or matric suction when \( p < 0 \). In unsaturated soils, water is held to the mineral grains by surface tension forces. When talking about infiltration, \( p \) and \( \psi \) will always be negative.

The relation between the pressure head \( \psi \) and water content \( \theta \) is called the moisture-characteristic curve. The relationship is highly nonlinear. Pressure head

![Figure 3: Dingman Figure 6-7 and 6-9](image)

is zero when water content equals porosity. There is a point when significant
volumes of air begin to appear in the soil and this is the air-entry tension $\psi_{ae}$. Beyond this point, water content begins to decrease rapidly and more gradually. After a certain value, even very large tensions will not dry out the soil because this water content is very tightly held in the soil pores by capillary and electrochemical forces. Given a certain degree of saturation, tension is much higher in finer-grained soils than in coarse grained soils.

In reality the value of tension at a given water content is not unique, but depends on the soil’s history of wetting and drying - however, this hysteresis is difficult to model and not usually incorporated in hydrologic models. Hysteresis effects on soil moisture properties:

1. Hysteresis implies that a one-to-one relation between $K_h$ or $\phi$ and $\theta$ does not exist with the history of soil wetting and drying playing a critical role.

2. Hysteresis is more common in the moisture characteristic curve $\phi(\theta)$ as compared to the hydraulic conductivity-water content relation.

3. Hysteresis is partly explained by the fact that during wetting of a soil, the filling of small diameter pores is aided by capillary forces, while during drying the same forces act to delay their emptying.

**Hydraulic Conductivity**

Hydraulic conductivity is the rate at which water moves through a porous medium under a unit potential energy gradient. Under saturated conditions, this size is determined by the soil-grain size. For unsaturated conditions it is determined by grain size and degree of saturation. $K$ is very low at low to moderate water content, and increases nonlinearly to its saturated value ($K^*$) as water content increases. $K$ increases by several orders of magnitude in going from clay to silty clay loam to sand (also depending on degree of saturation).

**i. Incorporating $K - \theta$ and $\psi - \theta$ Relations into Models** Brooks and Corey (1964), Campbell (1974) and Van Genuchten (1980) have proposed various relations for $K - \theta$ and $\psi - \theta$ relations:

$$|\psi(\theta)| = |\psi_{ae}| \left(\frac{\phi}{\theta}\right)^b$$  

(16)

$$K_h(\theta) = K_h^* \left(\frac{\theta}{\phi}\right)^c$$  

(17)
Figure 4: Dingman Figure 6-8

**Figure 6-8**

Soil-water pressure (tension), $\psi$, vs. degree of saturation, $S$, for soils of three different textures. Note that the vertical axis gives the base-10 logarithm of the absolute value of the pressure (which is negative), expressed in cm of water (cmH$_2$O). Curves are based on typical values given by Clapp and Hornberger (1978). The sandy-loam curve is for the soil discussed in Examples 6-1–6-3.
These equations ignore hysteresis, apply only to $|\psi| \geq |\psi_{ae}|$ - crude approximations for $|\psi| \leq |\psi_{ae}|$ can be made by a straight line from $|\psi| = 0$ to $|\psi| = |\psi_{ae}|$. $b$ is the pore size distribution index, $c$ is the pore-disconnectedness index.

$$c = 2b + 3$$

Typical values determined by statistical analysis of data for a large number of soils are given below:

![Table 6-1 Dingman](image)

**Table 6-1 Dingman**

<table>
<thead>
<tr>
<th>Soil Texture</th>
<th>$\phi$</th>
<th>$K_h$ (cm s$^{-1}$)</th>
<th>$\epsilon_{0,l}$ (cm)</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>0.395 (0.056)</td>
<td>$1.76 \times 10^{-2}$</td>
<td>12.1 (14.3)</td>
<td>4.05 (1.78)</td>
</tr>
<tr>
<td>Loamy sand</td>
<td>0.410 (0.068)</td>
<td>$1.56 \times 10^{-2}$</td>
<td>9.0 (12.4)</td>
<td>4.38 (1.47)</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>0.435 (0.086)</td>
<td>$3.47 \times 10^{-3}$</td>
<td>21.8 (31.0)</td>
<td>4.90 (1.75)</td>
</tr>
<tr>
<td>Silt loam</td>
<td>0.485 (0.059)</td>
<td>$7.20 \times 10^{-4}$</td>
<td>76.6 (51.2)</td>
<td>5.30 (1.96)</td>
</tr>
<tr>
<td>Loam</td>
<td>0.451 (0.076)</td>
<td>$6.95 \times 10^{-4}$</td>
<td>47.8 (51.2)</td>
<td>5.39 (1.87)</td>
</tr>
<tr>
<td>Sandy clay loam</td>
<td>0.420 (0.059)</td>
<td>$6.30 \times 10^{-4}$</td>
<td>29.9 (37.8)</td>
<td>7.12 (2.43)</td>
</tr>
<tr>
<td>Silty clay loam</td>
<td>0.477 (0.057)</td>
<td>$1.70 \times 10^{-4}$</td>
<td>35.6 (37.8)</td>
<td>7.55 (2.77)</td>
</tr>
<tr>
<td>Clay loam</td>
<td>0.476 (0.053)</td>
<td>$2.45 \times 10^{-4}$</td>
<td>63.0 (51.0)</td>
<td>8.32 (3.44)</td>
</tr>
<tr>
<td>Sandy clay</td>
<td>0.426 (0.037)</td>
<td>$2.17 \times 10^{-4}$</td>
<td>15.3 (17.3)</td>
<td>10.4 (1.64)</td>
</tr>
<tr>
<td>Silty clay</td>
<td>0.492 (0.064)</td>
<td>$1.03 \times 10^{-4}$</td>
<td>49.0 (62.1)</td>
<td>10.4 (1.45)</td>
</tr>
<tr>
<td>Clay</td>
<td>0.482 (0.050)</td>
<td>$1.28 \times 10^{-4}$</td>
<td>40.5 (39.7)</td>
<td>11.4 (3.70)</td>
</tr>
</tbody>
</table>

**Figure 5: Table 6-1 Dingman**

**Hydraulic Diffusivity** It is sometimes useful to use hydraulic diffusivity $D(\theta)$ as

$$D_h(\theta) = K_h(\theta) \frac{\partial \psi(\theta)}{\partial \theta}$$

(19)

notice the dimensions: $[m^2/s]$. This means that the flow due to the pressure gradient can be expressed as the product of the hydraulic diffusivity and the water-content gradient. Using the relationships above:

$$D_h(\theta) = -b\psi_{ae}K_h^\ast\phi^{-b-3}\theta^{b+2}$$

(20)

1c. **Field Capacity, Wilting Point and Available Water in a Soil Column**

**Soil field capacity** ($\theta_{fc}$) is the water content retained in a soil against gravity. It can be computed arbitrary by defining a reference pressure head at -340 cm
The field capacity is a function of soil texture and soil profile properties. For sands, \( \theta_{fc} \approx 0.1 \) while for clays \( \theta_{fc} \approx 0.3 \).

**permanent wilting point** \( \theta_w \) is the water content that limits plant transpiration, below which a plant will wilt. Wilting is a loss of turgor pressure inside the plant (water content within a plant is not sufficient to sustain plant stiffness). The wilting point water content can be computed arbitrary by defining a reference pressure head at -15000 cm (-1470 kPa):

\[
\theta_{pwp} = \phi \left( \frac{\phi_{ae}}{15,000} \right)
\]  

(22)

The wilting point soil is a function of soil texture, soil profile properties and vegetation characteristics. For sands, \( \theta_w \approx 0.05 \) while for clays \( \theta_w \approx 0.25 \)

**Available water for plant use**

\[
\theta_a = \theta_{fc} - \theta_w
\]

(23)

**Residual or hygroscopic water content** is the water remaining in the soil after drainage and evapotranspiration. It is a function of soil texture/type. Soils in nature don’t have water contents lower than this.
Figure 6: Table 6-13 Dingman