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1. Physics of Radiant Energy

All matter at a temperature above absolute zero radiates energy in the form of electromagnetic waves. A **blackbody** is a surface that completely absorbs all incident radiation. **The Planck Function** is an expression of the intensity of radiation emitted by a blackbody:

$$B_\lambda(T) = \frac{c_1\lambda^{-5}}{\pi(e^{c_2/\lambda T} - 1)} \tag{1}$$

The blackbody flux density or irradiance is obtained by integrating the Planck function over all wavelengths and is given by the **Stefan-Boltzmann Law**.

$$Q_r = \epsilon\sigma T^4 \tag{2}$$

where Q_r is the rate of energy emission per unit surface area per unit time $[EL^{-2}T^{-1}]$, T is the absolute temperature of the surface $[\Theta]$, σ is the universal *Stephan-Boltzmann constant* $[EL^{-2}T^{-1}\Theta^{-4}]$ $(\sigma = 5.67 \times 10^-8Wm^{-2K^{-4}}$. and ϵ is a dimensionless quantity called the **emissivity** of the surface, which ranges from 0 to 1. $\epsilon = 1$ for a **blackbody**, and most earth materials have emissivities close to 1.

The wavelength λ and frequency f of electromagnetic radiation are inversely related as:

$$\lambda f = c \tag{3}$$

where c is the speed of light $c = 2.998 \times 10^8 ms^{-1}$. The most important ranges in the earth's energy balance are the ultraviolet, visible and the infrared.

The wavelength at which the maximum energy radiation occurs λ_{max} is related to temperature via **Wien's Displacement Law**. We can obtain this expression by differentiating Planck's function and setting the derivative equal to zero.

$$\lambda_{max} T = 2897 \tag{4}$$

where λ_{max} is in μm and T is in K. While electromagnetic energy is transmitted through a vacuum undiminished, when it strikes matter, it interacts in the following 3 ways:

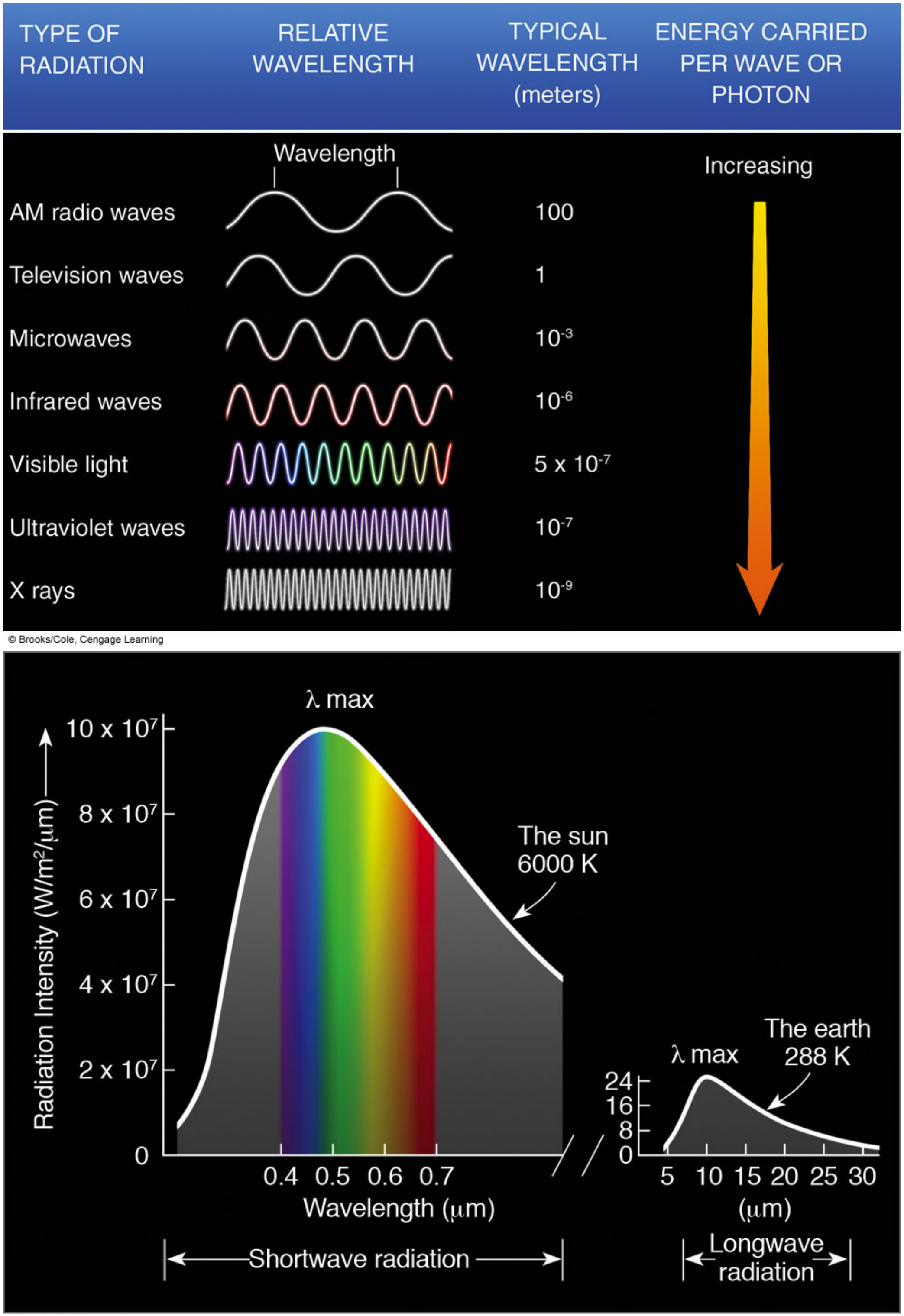
Absorptance $\alpha(\lambda)$ fraction of energy at wavelength λ absorbed by the surface, in turn raising its energy and/or causing a phase change.

Reflectance $\rho(\lambda)$ fraction of energy at wavelength λ reflected, this does not affect matter and continues undiminished. The reflectance integrated over the visible wavelengths is called the **albedo**

Transmittance $\tau(\lambda)$ fraction of energy at wavelength λ transmitted through the matter, does not affect the matter and continues to travel undiminished.

$$\alpha(\lambda) + \rho(\lambda) + \tau(\lambda) = 1 \tag{5}$$

The sun radiates approximately as a black-body with a temperature of 6000 K. As the energy enters, the gases in the earth's atmosphere are strong absorbers of energy at specific wavelenghts, so that energy is depleted in some portions. Oxygen and Ozone shield the earth from UV rays. Water vapor absorbs in the near infrared. Nearly all energy coming from the sun is less than 4 μm this energy is referred to as **solar radiation or shortwave radiation**. Radiation emitted by the sun approximates that emitted by a black body with maximum emitted at a wavelength of 0.48μm (calculated with Wiens Law). The solar spectrum is confined to .1 to 4 μm (hence shortwave). At the top of the earths atmosphere we define the solar irradiance Sc as the flux of solar radiation passing through a plane normal to the solar beam at the top of the atmosphere, with the Earth at its mean annual distance from the Sun, Sc = 1367Wm^{-2}, but this of course changes as the distance



TYPE OF RADIATION	RELATIVE WAVELENGTH	TYPICAL WAVELENGTH (meters)	ENERGY CARRIED PER WAVE OR PHOTON
AM radio waves		100	Increasing
Television waves		1	
Microwaves		10^{-3}	
Infrared waves		10^{-6}	
Visible light		5×10^{-7}	
Ultraviolet waves		10^{-7}	
X rays		10^{-9}	

Figure 1: Radiation characteristics and differences in radiation from the sun and from the earth. From Ahrens

changes.

100 units of incident energy, 26 reflected back (20 by clouds).
Clouds absorb 4 and gases about 16.
54 units reach the earth's surface.
Surface reflects 4
50 units are used for net radiant emission, sensible heat and latent heat.

The average temperature of the earth is $290K$, so the earth radiates approx. as a black body at this temperature. The earth mits 20 units of energy in the infrared range between 4 and 50 μm and this is referred to as **terrestrial** or **longwave** radiation. However, water vapor, carbon dioxide and other gases absorb radiation more strongly in the longwave part of the spectrum occupied by outgoing terrestrial radiation. Hence, incoming solar radiation passes through the atmosphere quite freely while terrestrial radiation is absorbed and reemitted in its upward passage through the atmosphere. This is called the **greenhouse effect**. Notice that the total incoming shortwave radiation is balanced by the total outgoing longwave radiation.

The transfer of latent heat via evaporation (mostly from oceans) adds another 24 units of energy to the atmosphere; *this is the largest source of atmospheric energy*. **Evapotranspiration** is a collective term for all the processes by which water in the liquid or solid phase becomes atmospheric water vapor. It includes evaporation of liquid water from rivers and lakes, bare soil and vegetative surfaces. Evaporation from within the leaves of plant and sublimation from ice and snow surfaces. 62% of the water that falls on the continents is evapotranspired. Of this 97% is from land surfaces and 3% from open water.

The energy balance equation can be written as follows:

$$\frac{\Delta Q}{\Delta t} = K + L - G - H - LE + A_w \tag{6}$$

2. Physics of Evaporation and Turbulent Energy Exchange

Air with temperature T_a lies above water with temperature T_s, some molecules will break the bonds in the liquid phase an escape. Some of the molecules in the air will re-enter the liquid phase..when an equilibrium is attained, the vapor pressure in the molecular layer immediately above the surface is the saturation vapor pressure at the temperature of the surface e_s*. The water vapor will be moved from the surface to the boundary layer via turbulent eddies.

2a. *Eddy Correlation Approach*

Eddy correlation measurements are often considered the "true" evaporation rates because the method has a sound theoretical foundation and requires no assumptions. It is based on the principle that turbulence is the primary mechanism to transport water vapor away from the surface into the atmosphere. Before defining the eddy correlation approach, let's remember some basic statistics, these will help us define the turbulent transport.

The Mean

Several types of averaging, including temporal $()^t$, spatial $()^x$ or ensemble mean $()^e$, are relevant for turbulent flow. If we define P as the averaging period where $\Delta t = P/N$, the time average can be expressed as:

$$\overline{A}^t = \frac{1}{N}\sum_{i=1}^{N} A(i,s) \tag{7}$$

Temporal averages are generally used at fixed-points such as towers. On the other hand, for moving observations such as aircraft measurements, the one-dimensional spatial average can be used. Over an averaging length L where $\Delta x = L/N$, the spatial average is defined as:

Variance and Standard Deviation

Variance is the dispersion of data about the mean.

$$\sigma_a^2 = \frac{1}{N-1}\sum_{i=0}^{N-1}(A_i - \bar{A})^2 \tag{8}$$

where σ_a is the *standard deviation*. When N is very large (as it is for most PBL studies) $N \approx N-1$, and because $a' = A - \bar{A}$,

$$\sigma_a^2 = \overline{a'^2} \tag{9}$$

and this is true for winds and scalars. Variables such as $\overline{u'^2}$, $\overline{q'^2}$, $\overline{\theta_v'^2}$, can be interpreted as variances.

Covariance and Correlation

Using $N \approx N-1$, we define the covariance as:

$$\begin{aligned} covar(A,B) &= \frac{1}{N}\sum_{i=0}^{N-1}(A_i - \bar{A})(B_i - \bar{B}) \\ &= \frac{1}{N}\sum_{i=0}^{N-1} a_i' b_i' \\ &= \overline{a'b'} \end{aligned} \tag{10}$$

The rate of upward movement of water vapor is proportional to the time average of the product of the instantaneous fluctuations of vertical air movement u_a' and of specific humidity q' around their mean values.

$$E = \frac{\rho_a}{\rho_w}\overline{u_a' q'} \tag{11}$$

Unfortunately the type of instrumentation required for eddy correlation is quite difficult to attain. For this reason there are some simplification that can be applied to the theory in order to use more widely available measurements.

The rate of evaporation is the rate at which molecules move from the saturated surface layer into the air above, and it is proportional to the difference between the vapor pressure of the surface layer and the vapor pressure of the overlying air e_a.

$$E \propto e_s * - e_a \tag{12}$$

We can model evaporation as a diffusive process that follows Fick's first law. Diffusion is the process by which constituents of a fluid, such as momentum, heat or a dissolved or suspended constituent are transferred due to differences in concentrations in the constituent in different parts of the fluid. The rate of transfer of a constituent X in the direction z is given by:

$$F_z(X) = -D_x \frac{dC(X)}{dz} \tag{13}$$

There $F_z(X)$ is the rate of transfer of X in the direction z per unit area per unit time (called the flux of X), $C(X)$ is the concentration of X and D_x is called the **diffusivity**. The minus sign indicated that X always moved from regions of higher to lower concentrations. The value of D_x depends on the nature of the property and the physical mechanism by which it is transferred.

We can also write evaporation in finite-difference form as:

$$E = K_E v_a (e_s - e_a) \tag{14}$$

where E is the evaporation rate $[LT^{-1}]$ and e_s and e_a are the vapor pressures of the evaporating surface and overlying air, v_a is the wind speed and K_E is a coefficient that reflects the efficiency of vertical transport of water vapor by turbulent eddies of wind. We can express K_E for a neutral atmosphere as:

$$K_E = \frac{0.622 \rho_a k^2}{P \rho_w \left[ln\left(\frac{z_m - z_d}{z_0} \right) \right]^2} \tag{15}$$

where ρ_a is the density of air, P is the atmospheric pressure $k = 0.4$ is the Von Karman constant, z_m is the height at which wind speed and air vapor pressure are measured. z_d is the zero-plane displacemnt height and z_0 is the roughness height, these both depend on the roughness of the surface, which determines the intensity of the turbulent eddies. Modifications depending on the stability of the atmosphere can be accounted for.

2b. *Latent Heat Exchange*

Evaporation is always accompanied by a transfer of latent heat from the evaporating body into the air. To find the rate of latent-heat transfer we simply multiply the evaporation rate by the latent heat of vaporization λ_v and the mass density of water ρ_w.

$$LE = \rho_w \lambda_v E = \rho_w \lambda_v K_E v_a (e_s - e_a) \tag{16}$$

The latent heat of vaporization is a function of temperature:

$$\lambda_v = 2.5 - 2.36 \times 10^{-3} T \tag{17}$$

where λ_v is in $MJkg^{-1}$ and T is in $^{\circ}C$

2c. *Sensible Heat Exchange*

The upward rate of sensible-heat exchange by turbulent transfer is:

$$H = K_H v_a (T_s - T_a) \tag{18}$$

where

$$K_H = \frac{c_a \rho_a k^2}{\left[ln \left(\frac{z_m - z_d}{z_0} \right) \right]^2} \tag{19}$$

where c_a is the heat capacity of dry air is 1005 $Jkg^{-1\circ}C^{-1}$. Again, we can adjust for non-neutral atmospheric conditions.

2d. *Bowen Ratio*

The ratio of the sensible heat to latent heat exchange is called the Bowen ratio

$$B = \frac{H}{LE} \tag{20}$$

combining the expressions for both fluxes we have:

$$B = \frac{c_a P (T_s - T_a)}{0.622 \lambda_v (e_s - e_a)} = \gamma \frac{(T_s - T_a)}{(e_s - e_a)} \tag{21}$$

where

$$\gamma = \frac{c_a P}{0.622 \lambda_v} \tag{22}$$

This is called the **psychrometric constant** (which is not really a constant). We can also define the **evaporative fraction**, EF as the ratio of latent heat to total turbulent heat exchange.

$$EF = \frac{LE}{LE + H} = \frac{1}{B + 1} \tag{23}$$

3. Free Water Evaporation

Is referred to as evaporation that would occur from an open water surface in the absence of advection and changes in heat storage.

3a. *Water-Balance Approach and Energy-Balance Approach*

From the water-balance approach, we can write the following conservation equation

$$E = W + SW_{in} + GW_{in} - SW_{out} - GW_{out} - \Delta V \tag{24}$$

From the energy-balance approach, we can write the following equation:

$$E = \frac{K + L - G - H + A_w - \Delta Q/\Delta t}{\rho_w \lambda_v} \tag{25}$$

Using either of these equations, if we know all the fluxes on the right-hand side, we can compute E. The problem is that any errors in the calculation of the fluxes will corrupt our estimate of E, and that it is very difficult (particularly for the water balance approach), to calculate all the fluxes.

3b. *Mass Transfer Approach*

The mass transfer approach makes direct use of the equation presented above:

$$E = \frac{0.622 \rho_a k^2}{P \rho_w \left[ln \left(\frac{z_m - z_d}{z_0} \right) \right]^2} v_a (e_s - e_a) \tag{26}$$

and while the equation is sometimes simplified, this version of the equation is useful when we will derive the Penman equation. It must be emphasized that this equation gives the instantaneous rate of evaporation, and average quantities should not be used to calculate an average E.

3c. *Penman Combination Equation*

From equation 24, if we neglect ground heat conduction, water advected energy and change in energy storage:

$$E = \frac{K + L - G - H}{\rho_w \lambda_v} \tag{27}$$

The sensible heat is given by 18, and we can define the slope of the saturation vapor pressure curve between the surface and a point very near the ground as:

$$\Delta = \frac{e_s * - e_a *}{T_s - T_a} \tag{28}$$

substituting in the sensible heat equaiton:

$$\begin{aligned}
H = \frac{K_H v_a (e_s * - e_a *)}{\Delta} &= \frac{K_H v_a (e_s * - e_a)}{\Delta} - \frac{K_H v_a (e_a * - e_a)}{\Delta} && (29)\\
&= \frac{K_H v_a}{\Delta} \frac{E}{K_E v_a} - \frac{K_H v_a (e_a * - e_a)}{\Delta} && (30)\\
& && (31)
\end{aligned}$$

$$E = \frac{K + L - G + \frac{K_H v_a}{\Delta}(e_a * - e_a)}{\rho_w \lambda_v + \frac{K_H}{\Delta K_E}} \tag{32}$$

since $K_H / K_E = \gamma \rho_w \lambda_v$

$$\begin{aligned}
E &= \frac{\Delta(K + L - G) + K_H v_a (e_a * - e_a)}{\rho_w \lambda_v (\Delta + \gamma)} && (33)\\
&= \frac{\Delta(K + L - G) + \gamma \rho_w \lambda_v K_E v_a (e_a * - e_a)}{\rho_w \lambda_v (\Delta + \gamma)} && (34)
\end{aligned}$$

This is called the **Penman equation or combination equation**. Notice that the essence of the Penman equation is that evaporation ratio is a weigthed sum of a rate due to net radiation and a rate due to mass transfer. And the mass transfer depends on the vapor pressure and saturation vapor pressure in the air. Because of its satisfactory results, and good theoretical foundation, the combination method has become the standard hydrological method for determining free water evaporation.

3d. *Pan Evaporation Approach*

A direct approach to determine free-water evaporation is to expose a cylindrical pan of liquid water to the atmosphere and to solve the following equation $E = W - [V_2 - V_1]$. Where W is precipitation during the time Δt and V_1 and V_2 are the storages at the beginning and end of the period. Pans are placed on clearings.