Plane parallel non-scattering atmosphere

Solution to EvZ (8-58)

$$\mu \frac{dI}{d\tau} = I(\tau,\mu) - \psi(\tau,\mu) \tag{1}$$

where μ is cosθ. This is a linear first order differential equation of the form

$$\frac{dy}{dx} + P(x)y = q(x) \tag{2}$$

with solution

$$y = \frac{\int u(x)q(x)dx + C}{u(x)} \tag{3}$$

where $u(x) = \exp\left(\int P(x)dx\right)$. Rearranging EvZ's equation to match the form of (2) yields

$$\frac{dI}{d\tau} - \frac{1}{\mu} I(\tau,\mu) = -\frac{1}{\mu}\psi(\tau,\mu) \tag{1a}$$

Matching (1a) and (2) have $x = \tau$, $y = I$, $P(\tau) = -1/\mu$, $q(\tau) = -\psi/\mu$ and $u(\tau) = \exp(-\tau/\mu)$. The solution is

$$I(\tau,\mu) = \frac{\int_{\tau_0}^{\tau} -e^{-\eta/\mu}\frac{\psi}{\mu}d\eta + C}{e^{-\tau/\mu}} = \frac{1}{\mu}\int_{\tau}^{\tau_0} e^{(\tau-\eta)/\mu}\psi(\tau,\mu)d\eta + Ce^{\tau/\mu}$$

which is the same as EvZ's solution (8-59):

$$I(\tau,\mu) = Ae^{\tau/\mu} + \frac{1}{\mu}\int_{\tau}^{\tau_0} \psi(\eta,\mu)e^{(\tau-\eta)/\mu}d\eta \tag{4}$$

Now how do we use this. τ is defined as 0 at the surface and τ_m at the top of the atmosphere.

Downward radiation: For *downward* radiation through the atmosphere, we ignore the 3K cosmic background so there is no source term outside the atmosphere so the $Ae^{\tau/\mu}$ term is 0.

$$I(\tau,\mu) = 0 + \frac{1}{\mu}\int_{\tau}^{\tau_m} \psi(\eta,\mu)e^{(\tau-\eta)/\mu}d\eta \tag{5}$$

This represents the summation of a downward source at every optical depth level, η, in the atmosphere starting at τ_m at the atmospheric top down to the level, τ. The emission from each level is then attenuated by an amount $\exp(-[\eta-\tau]/\mu)$.

Upward radiation: For radiation passing *upward* through the atmosphere, μ becomes $-\mu$. There is also a surface radiation term. The surface is defined at $\tau = 0$.

$$I(\tau,-\mu) = I(0,-\mu)e^{-\tau/\mu} + \frac{1}{\mu}\int_0^{\tau} \psi(\eta,-\mu)e^{-(\tau-\eta)/\mu}d\eta \tag{6}$$

If the observing point is above the atmosphere like a satellite,

$$I(\tau_m,-\mu) = I(0,-\mu)e^{-\tau_m/\mu} + \frac{1}{\mu}\int_0^{\tau_m} \psi(\eta,-\mu)e^{-(\tau_m-\eta)/\mu}d\eta \tag{7}$$

Note error in EvZ eq. (8.62). dμ should be dη.