Deriving Precipitable Water Vapor (PWV) from GPS receivers on the ground

Precipitable water (PW) is the amount of water in a column of atmosphere. It is usually measured as the height of a layer of water if that water were condensed to liquid. PW includes all the water in the column, vapor and condensed. Precipitable water vapor (PWV) is the total water vapor in the column. It turns out that signals from the Global Positioning System (GPS) satellites are affected by water vapor and they can therefore be used to measure the amount of water vapor in the atmosphere.

The GPS is the first Global Navigation Satellite System (GNSS). It is a US Air Force precise positioning system with 24-32 satellites in 12 hour orbits. There are other GNSS systems: the Russian GLONASS (exists), the European Galileo (under development) and Chinese COMPASS (under development) systems.

The GPS satellites continuously radiate continuous wave (CW) (that is non-pulsed) signals at several frequencies:

- L1 (1575.42 MHz): Mix of Navigation Message, coarse-acquisition (C/A) ranging code (300 m resolution or chip length) and encrypted precision P(Y) ranging code (30 m resolution or chip length).
- L2 (1227.60 MHz): P(Y) code, plus the new L2C (civilian ranging code) on the Block IIR-M and newer satellites.
- L3 (1381.05 MHz): Used by the Nuclear Detonation (NUDET) Detection System Payload (NDS) to signal detection of nuclear detonations and other high-energy infrared events. Used to enforce nuclear test ban treaties.
- L4 (1379.913 MHz): Being studied for additional ionospheric correction.
- L5 (1176.45 MHz): Proposed for use as a civilian safety-of-life (SoL) signal (see GPS modernization). This frequency falls into an internationally protected range for aeronautical navigation, promising little or no interference under all circumstances. The first Block IIF satellite that would provide this signal is set to be launched in 2009.

These signals are used to determine the positions of objects such as cars, aircraft and satellites.

Ionospheric correction

The reason for the original L1 and L2 signals was to isolate and remove the effects of the ionosphere which would otherwise adversely affect the precise positioning. A version of the refractivity equation for frequencies below 300 GHz is shown in (0).

$$N = (n-1) = k_P \frac{P}{T} + k_{H2O} \frac{P_w}{T^2} - k_{ion} \frac{n_e}{f^2} + k_{liq} W_{liq} + k_{ice} W_{ice} \qquad (0)$$

where N is refractivity, n is the index of refraction = c/v where c is the speed of light in a vacuum and v is the speed of light in the medium, P is atmospheric pressure, T is atmospheric temperature in K, P_w is the partial pressure of water vapor, n_e is the number density of free electrons, W_{liq} is the mass density of liquid water suspended in the atmosphere and W_{ice} the mass density of ice suspended in the atmosphere and the k's are scale constants. In the context of the ionosphere, the key term is the ionospheric term which is dispersive because it depends on the signal frequency, specifically the inverse of the signal frequency squared. As a result, by measuring the propagation delay through the ionosphere at at least 2 frequencies, the ionospheric effect can be isolated and removed.

The problem for civilian users was that originally the only L2 signal available from the GPS satellites was modulated by an encrypted P(Y) code that unclassified GPS receivers could not demodulate. There were some tricks implemented based on correlation between the L1 P(Y) modulation and the L2 P(Y) modulation that provided some ionospheric information at lower signal to noise ratio. The new L2C signal is an unencrypted, civilian-accessible ranging code modulation that allows civilians access to the L2 signal and a precise ionospheric correction.

The military and NOAA are very interested in using GPS to monitor and predict the state of the ionosphere. This field is called "space weather". There are embryonic upper atmosphere models coupled to data assimilation systems that are being developed to improve predictions at ionospheric altitudes.

Remote sensing PWV via GPS

From the standpoint of remotely sensing the atmosphere, the GPS signals are useful for remotely because the atmosphere causes the GPS signals to slow down and therefore take longer to propagate than it would in a vacuum. When measured by GPS receivers on the ground, this effect can be used to determine the amount of water vapor in the atmosphere.

Consider the time it takes a signal to propagate from a GPS satellite to a GPS receiver. If the geometric distance is L, and the index of refraction, $n=c/v$, is 1, then the time is $L/v = Ln/c$. If the index of refraction varies along the path, then the propagation time is

$$t = \int_L \frac{dl}{v} = \frac{1}{c}\int_L n\,dl \tag{1}$$

In the atmosphere, n is almost 1. So there is a term called refractivity, N, which is

$$N = (n\text{-}1)*10^6 = c_1\,\mathrm{n}_d + c_2\,\mathrm{n}_w + c_3\,\mathrm{n}_e + c_4\,\mathrm{n}_p \tag{2}$$

where $c_2\, n_w = (c_{w1} + c_{w2}/\mathrm{T})\,\mathrm{n}_w$. n_e is the density of free electrons in the ionosphere which we eliminate by using the measuring at the 2 GPS frequencies because c_4 is proportional to $1/f^2$. n_p is a very small contribution from atmospheric particles and we ignore it for now. Using the ideal gas law, we can rewrite (2) as

$$N = (n\text{-}1)*10^6 = k_1 *P_d/T + (k_2 *P_w/T + k_3 *P_w/T^2) \tag{3}$$

where Pd is the dry partial pressure. When pressure is in mb and temperature in K, k_1 = 77.6890 N-units K/mb, k_2 = 71.2952 N-units K/mb and k_3 = 375463 N-units K^2/mb. The first term on the right is often written as N_d. The accuracy of the N_d term is of 0.02% of N_d. ***NOTE: There have been recent discussions about this and the k_1 constant needs to be re-measured***. The k_1 term accounts for the contribution of 375 ppm of CO_2 in the atmosphere and is slightly changing over time. The accuracy of the N_w term is of 0.2%.

Note that this equation is often written as two terms:

$$N = (n\text{-}1) = 77.6\ P/T + 3.73\mathrm{e}5\ P_w/T^2. \tag{4}$$

This is an approximation to the 3 term equation above.

Assuming the ideal gas law, $P/T = n\,R^*$ where n is the number density of the atmosphere in moles per cubic meter then we can rewrite (3) as

$$N = (n\text{-}1)*10^6 = [\,k_1\, n_d + (k_2\, n_w + k_3\, n_w/T)]\ R^*/100 \tag{5}$$

because 1 mb = 100 Pa.

If the atmosphere is plane parallel, then the path through the atmosphere is $dl = dz / \cos\theta$ where θ is the zenith angle. (NOTE: we are ignoring any bending). The propagation time is

$$t = \frac{1}{c}\int_L n\, dl = \frac{1}{c}\int_L \left(1 + \frac{N}{1e6}\right) dl = \frac{L}{c} + \frac{1}{c}\int_{atmos} \left(\frac{N}{1e6}\right) \frac{dz}{\cos\theta} \tag{6}$$

The non-geometric delay term is

$$t_{NG} = \frac{1}{c}\int_{atmos} \left(\frac{N}{10^6}\right) \frac{dz}{\cos\theta} = \frac{R}{c}\int_{atmos} \left(\frac{k_1 n_d + n_w\left[k_2 + k_3/T\right]}{10^8}\right) \frac{dz}{\cos\theta} \tag{7}$$

The surface pressure is given by the hydrostatic integral

$$P = \int_0^\infty g\rho dz = \int_0^\infty g\left[n_d m_d + n_w m_w\right] dz \tag{8}$$

where m_d and m_w are the mean molar mass of dry molecules and water molecules respectively. Assume g is a constant so we can pull g outside of the integral. Then we can rewrite (8) as

$$\int_0^\infty n_d dz = \frac{P}{g m_d} - \int_0^\infty n_w \frac{m_w}{m_d} dz \tag{9}$$

and

$$t_{NG} = \frac{R}{c\,10^8}\left(\frac{k_1 P}{g m_d \cos\theta} + \int_0^\infty \frac{dz}{\cos\theta} n_w \left[-k_1 \frac{m_w}{m_d} + k_2 + \frac{k_3}{T}\right]\right) \tag{10}$$

We define the "wet delay", t_w, as

$$t_w = \frac{R}{c\,10^8}\left(\int_0^\infty n_w dz \left[-k_1 \frac{m_w}{m_d} + k_2 + \frac{k_3}{T}\right]\right) = \cos\theta\, t_{NG} - \frac{R}{c\,10^8}\frac{k_1 P}{g m_d} \tag{11}$$

From (11), we can isolate the precipitable water vapor, PWV, as

$$PWV = \int_0^\infty n_w dz = \frac{\left(\frac{c\,10^8 \cos\theta}{R} t_{NG} - \frac{k_1 P}{g m_d}\right)}{\left[-k_1 \frac{m_w}{m_d} + k_2 + \frac{k_3}{\overline{T}}\right]} \tag{12}$$

where $$\overline{T} = \frac{\int_0^\infty n_w dz}{\int_0^\infty \frac{n_w}{T} dz} \tag{13}$$

Written in this way, PWV is in units of moles/m^2. Multiplying PWV by 0.018 kg of water per mole gives kg/m^2 which is also equal to precipitable mm because condensed water has a density of 1000 kg/m^3.

Note that to derive accurate estimates of PWV from GPS signals we must have

- a very accurate estimate of the air pressure at the height of the GPS receiver antenna
- a good estimate of the air temperature over the lower troposphere where most of the atmospheric water resides.

Therefore GPS receivers used for PWV estimation are usually collocated with a surface meteorological instrumentation package near by. If there is not much topography, then data from a more remote surface meteorology station can be used. For instance, there is

Web pages of GPS PWV data:
http://www.atmo.arizona.edu/products/gps/gps_pwv.html
http://www.suominet.ucar.edu/
http://www.gpsmet.noaa.gov/jsp/index.jsp

individual noaa sites
http://gpsmet_test.fsl.noaa.gov/cgi-bin/gnuplots/rti.cgi