April 14, 2009

Locating Lightning

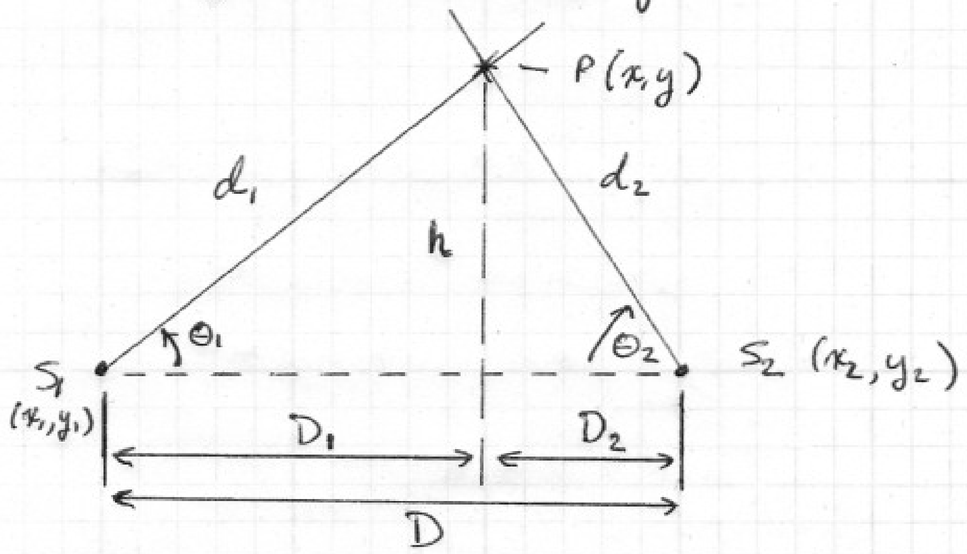
- What things can be located?
  - must have a definable location
  - must "emit" a signal
    - EM: near-DC to light
    - Pressure wave: near-DC to ultrasound

- How do we indicate location?
  - Point to it (direction finding)
  - image it in a plane (camera in space)
  - Compare: arrival times
    signal amplitudes

- Time-of-arrival (TOA) => wide spacing (wrt λ)

- Interferometry => narrow spacing

Location Using Direction-Finding



$$\left.\begin{aligned} D_1 &= d_1 \cos(\theta_1) \\ D_2 &= d_2 \cos(\theta_2) \end{aligned}\right\} \Rightarrow \underline{D = d_1 \cos(\theta_1) + d_2 \cos(\theta_2)}$$

$$h = d_2 \sin(\theta_2) = d_1 \sin(\theta_1)$$

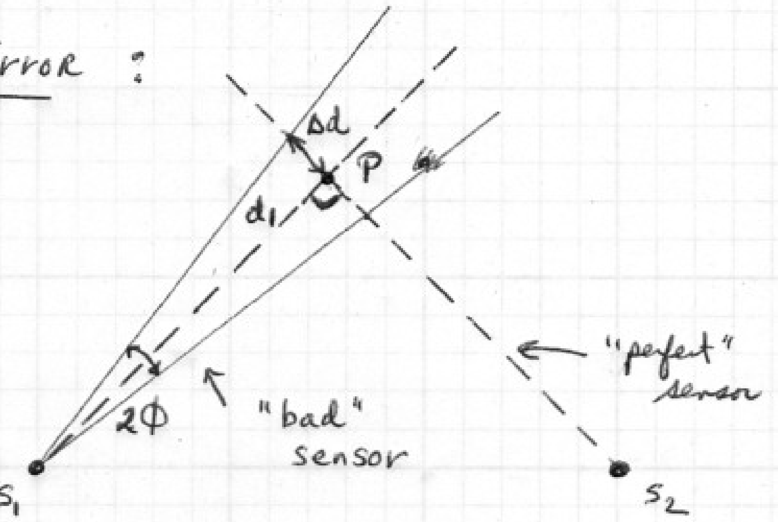
$$\Rightarrow \underline{d_1 \sin(\theta_1) - d_2 \sin(\theta_2) = 0}$$

- Know:
  D, θ_1 and θ_2

- Solve for:
  d_1 and d_2, then compute h

  - somewhat more complex on an oblate spheroid ...

Triangulation Error :

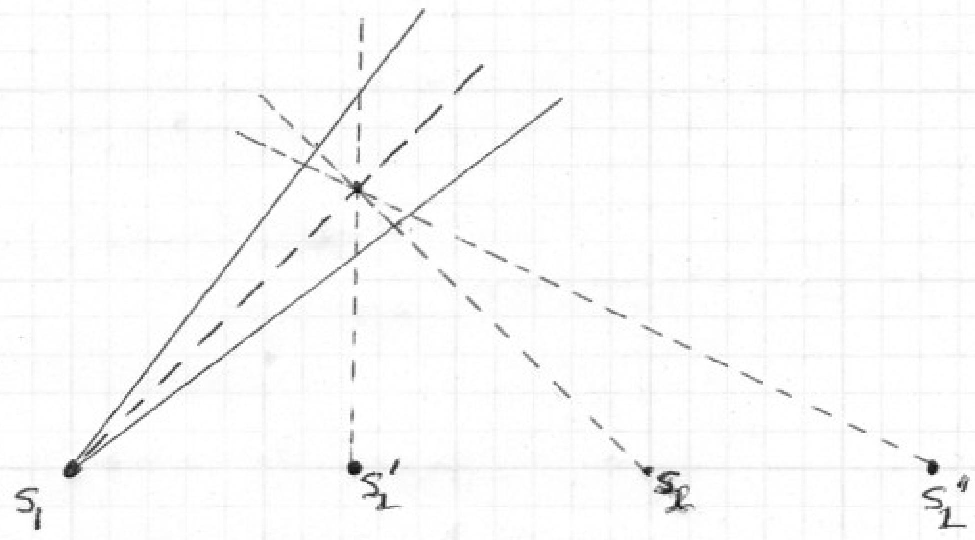


Azimuth error is
$\pm \Phi$ (small)

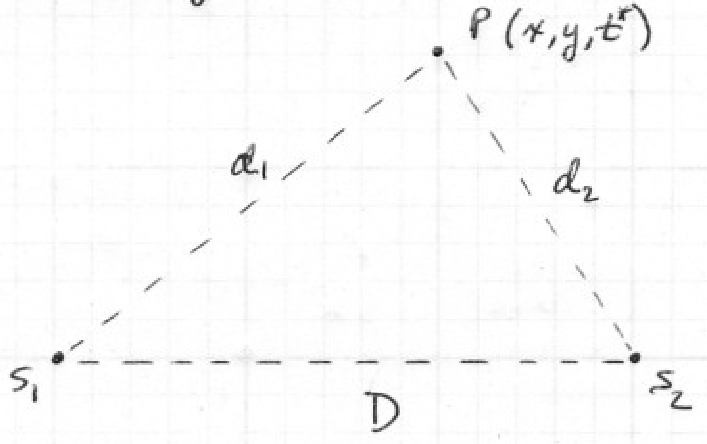
- given distance from S_1 to $P = d_1$; $\perp$ intersection
  the distance error (Δd):

$$\underset{(min.)}{\Delta d} = d_1 \sin(\phi) \qquad {}^{*}\left(\text{error} \propto \underline{\underline{d}} \Rightarrow \propto \underline{\underline{D}}\right)$$

- for different intersection angles,
  Δd is <u>larger</u>



Location using time-of-arrival



- precise time of "event" @ $p(\cdot) = t^*$
  - Signal arrives @ S_2 at time t_1
  - " " @ S_2 at " t_2

$$t_1 = t^* + d_1/v \qquad (v = \text{propagation speed})$$
$$t_2 = t^* + d_2/v$$

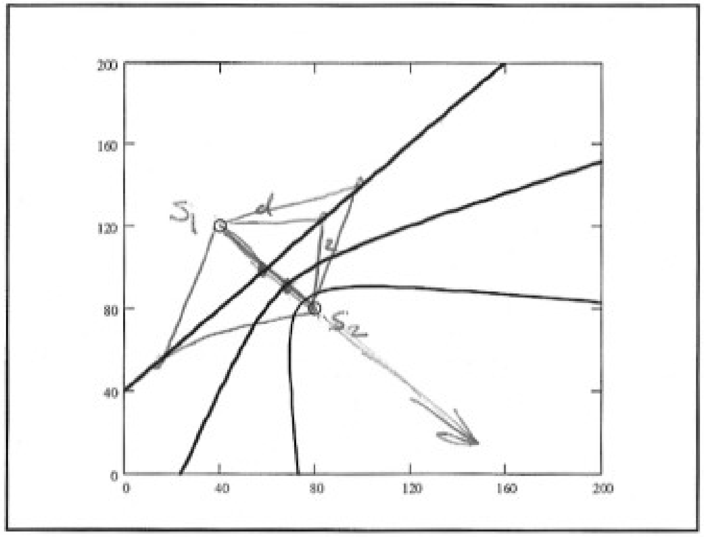
$$t_1 - t_2 = \frac{d_1 - d_2}{v}$$

- If we only know t_1 and t_2, then we must identify all pts satifying

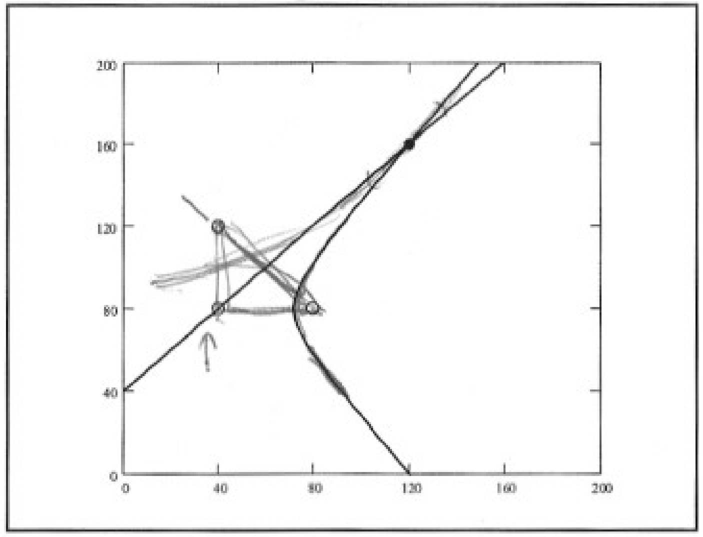
$$t_1 - t_2 = \frac{d_1 - d_2}{v} = \text{CONSTANT}$$

$\Rightarrow$ hyperbola w/ foci @ S_1 and S_2

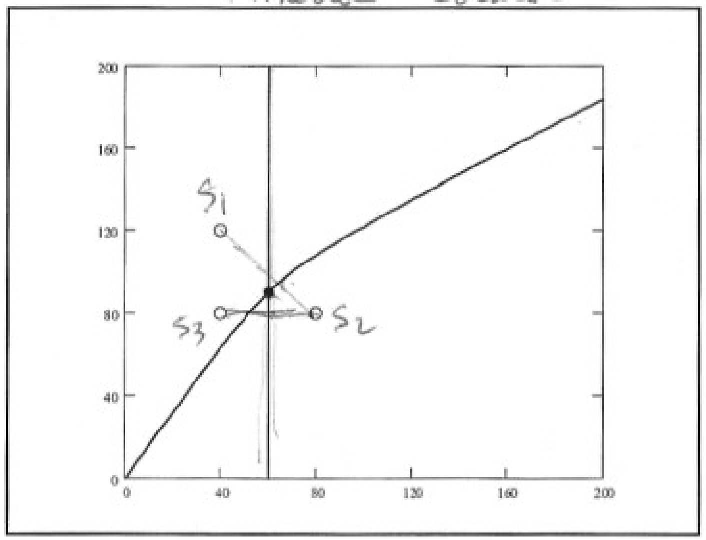
2 sensor locus of pts

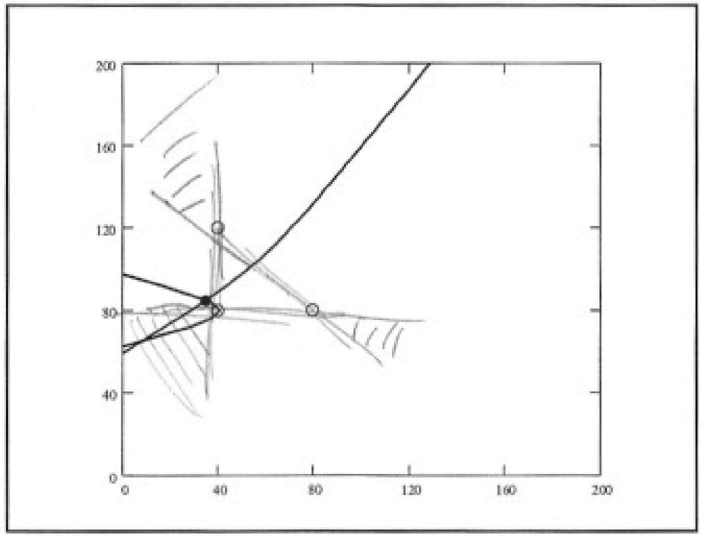


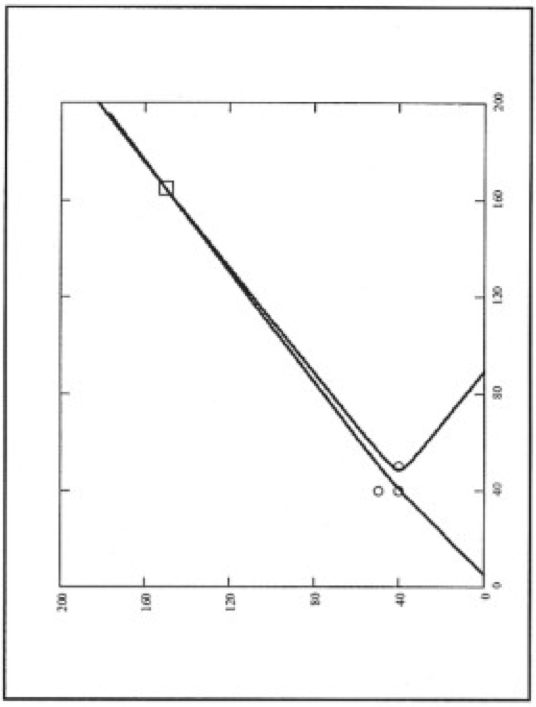
Exterior



interior Locator






200
160
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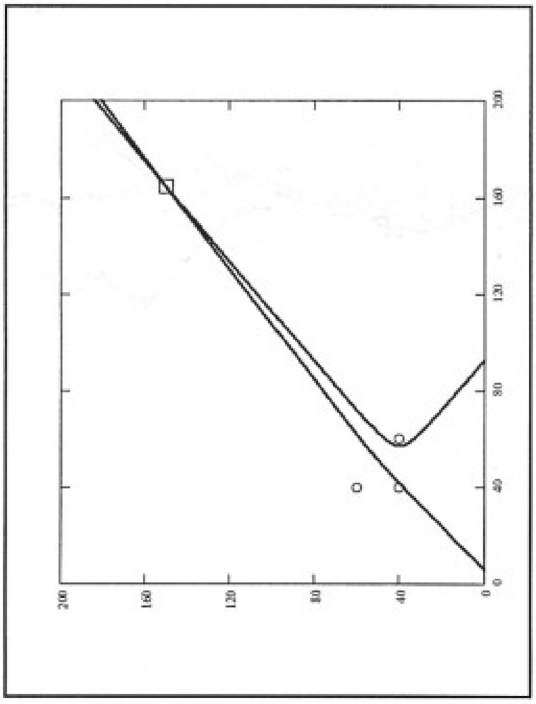
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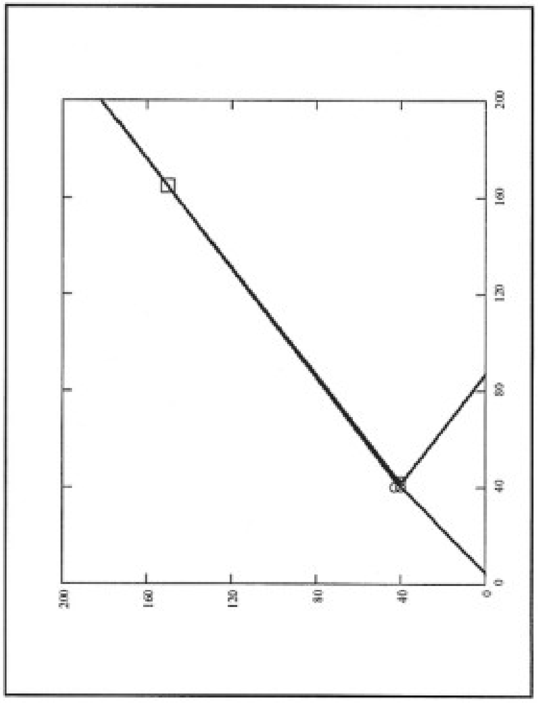
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200

time-of-arrival error :

to first order, errors are due to toa measurement error:

$$\hat{t}_i = t_i + n$$

added random error

$$E(n) = 0 \qquad E(n^2) = \sigma_n^2$$

$\therefore$ the time-difference measurement is

$$\hat{t}_1 - \hat{t}_2 = (t_1 - t_2) + (n_1 - n_2)$$

$$E(n_1 - n_2) = 0 \qquad E\left((n_1 - n_2)^2\right) \leq 2\sigma_n^2$$

$(= 2\sigma_n^2$ if n_1 & n_2 uncorr.$)$

- given RMS timing error of $1.41\sigma_n$, $(\sqrt{2})$?
the position uncertainty is

$(1.41\sigma_n)v \Rightarrow$ independent of sensor spacing

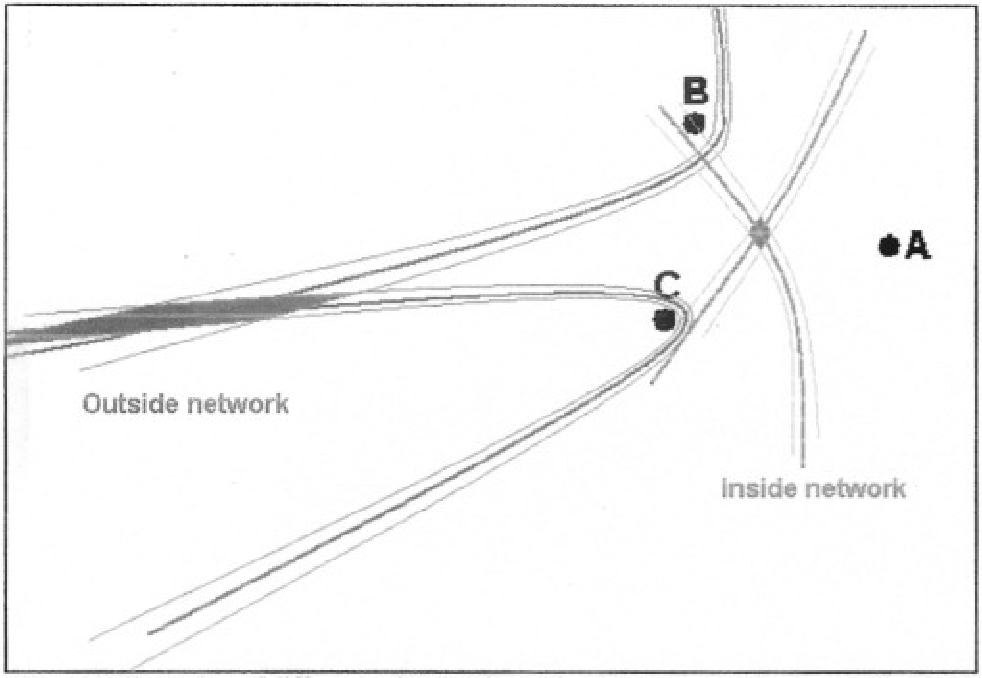

Figure 6 Illustration of differences in TOA-based location accuracy for sources located inside and outside a 3-sensor network (locations A-C).

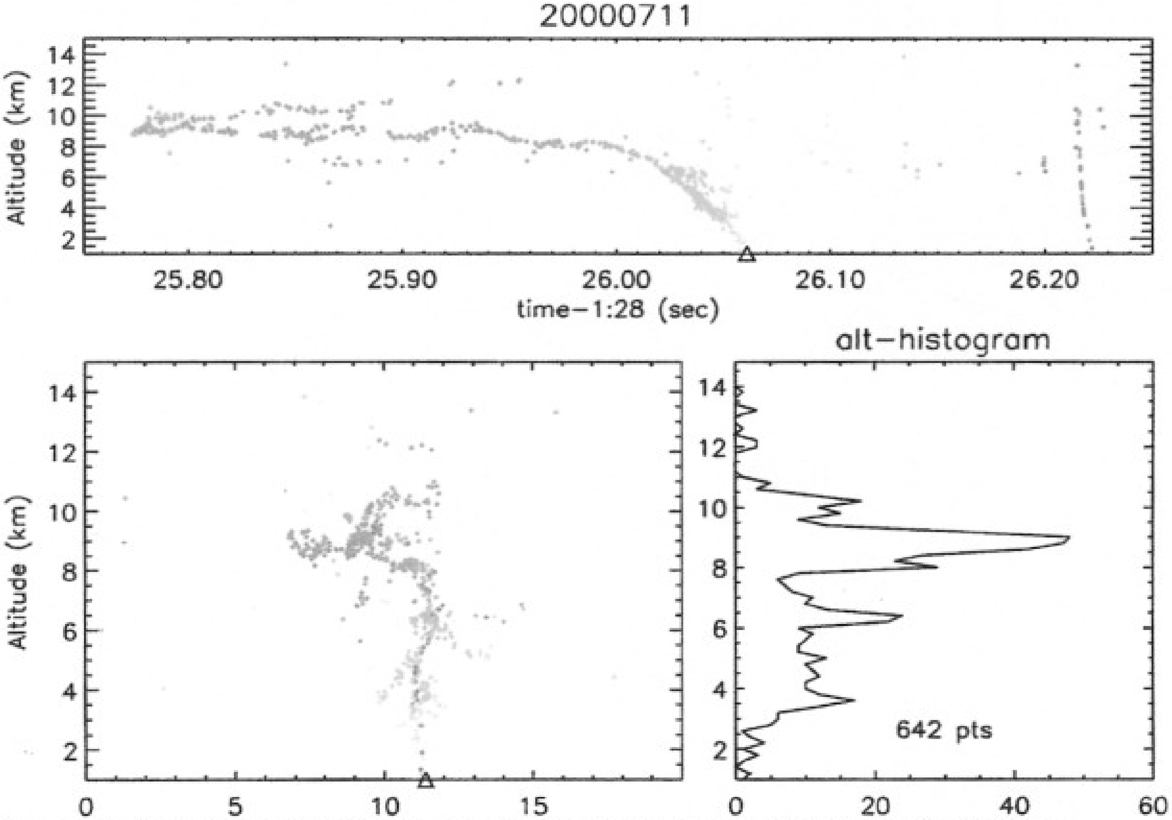

Figure 4. "Bolt from the blue" cloud-to-ground flash detected using VHF time-of-arrival lightning mapping. See text for details.