Mesoscale Meteorological Modeling: Spring 2008
Homework #4

This assignment investigates numerical methods to represent the advection equation. For purposes of the assignment, this equation can be written as:

$$\frac{\partial \phi}{\partial t} = -U \frac{\partial \phi}{\partial x}$$

Where ϕ is a generic variable, U is a constant speed of advection, t is the time dimension, and x is the space dimension.

<u>Questions:</u>

1. Consider the following fourth order space leapfrog numerical advection scheme (p. 346 in Pielke text)

$$\frac{\phi_i^{\tau+1} - \phi_i^{\tau-1}}{\Delta t} = -\frac{U}{\Delta x}\left[\frac{4}{3}\left(\phi_{i+1}^{\tau} - \phi_{i-1}^{\tau}\right) - \frac{1}{6}\left(\phi_{i+2}^{\tau} - \phi_{i-2}^{\tau}\right)\right]$$

Evaluate the linear stability of this advection scheme using Von Neumann's method. What expressions do you get for the amplitude and phase error? Show your work.

Using these expressions, produce a table of amplitude and phase errors for this scheme identical to Table 10-1 in the Pielke text (p. 290). How does the numerical behavior of this scheme compare with the second order in space leapfrog scheme discussed in class? Does a higher order scheme lead to improvement in representation of amplitude and phase?

2. In this part, you will program several different advection schemes and integrate these schemes forward in time. To initialize the simulation, assume a periodic function, such as a cosine wave. Use cyclic boundary conditions. For each scheme, design a programming strategy to compute amplitude and phase errors for the various Courant numbers and wavelengths as in Table 10-1. Display these results similarly as a table. (Hints: Your solutions will not be exactly as in Table 10-1, but should be fairly close. The programming strategy for phase error is the most challenging part.)

The schemes are:

a) Forward upstream (for positive U):

$$\frac{\phi_i^{\tau+1} - \phi_i^{\tau}}{\Delta t} = -\frac{U}{\Delta x}\left(\phi_i^{\tau} - \phi_{i-1}^{\tau}\right)$$

b) Second order in space leapfrog:

$$\frac{\phi_i^{\tau+1} - \phi_i^{\tau-1}}{2\Delta t} = -\frac{U}{2\Delta x}\left(\phi_{i+1}^{\tau} - \phi_{i-1}^{\tau}\right)$$

c) Fourth order in space leapfrog from Problem #1. For this one, you will compare with your own computed values.

Note: For the leapfrog schemes, you can use forward upstream differencing for first few time steps. You will also get unphysical computational mode solutions.

Discuss your results. Emphasize the caveats of representing numerical advection with a finite difference approach.

3. For the leapfrog schemes in problem #2, repeat your numerical integrations with the incorporation of the Asselin filter discussed in class and in Robert Fovell's notes. Does use of this filter improve amplitude and phase errors? What is the sensitivity to the weighting parameter ε? Why is it standard practice to have such a filter when implementing a leapfrog scheme in a numerical model?

4. For the regional model you evaluated in Assignment #2, how is advection numerically discretized? What are the characteristics of the advection scheme, in terms of amplitude and phase error, and how do these compare with some of the approaches we discussed in class? If it is a complicated higher order scheme it should definitely be better to justify the additional computational expense! You may have to dig a bit in the literature to find this information, as it is probably not explicitly stated in the model documentation.