1. Saturated Adiabatic Processes

If vertical ascent continues above the LCL, condensation will occur and the latent heat of phase change will be released. This latent heating will cause the temperature to decrease more slowly with pressure above the LCL than below. This means that the lapse rate above the LCL will be smaller than the lapse rate below the LCL. To describe the behavior of the parcel we use a **Pseudoadiabatic process or irreversible saturated adiabatic process** where all the condensed water or sublimated ice falls out of the air parcel as soon as it is produced (if we were to include the presence of the condensate it would unnecessarily complicate the process representation). Because the water component accounts for only a small fraction of the system's mass, the pseudo adiabatic process is nearly identical to a reversible saturated adiabatic process. Through any point of an aerological diagram there is one, and only one, pseudoadiabat. During this process, the change of water vapor mixing ratio is given by $dw = dw_s$

1) DERIVATION OF THE EXPRESSION FOR PSEUDOADIABATIC LAPSE RATE

Our objective is to find how the temperature changes as we move up the pseudoadiabat ($\Gamma_s = -dT/dz$). We begin with the First Law of Thermodynamics:

$$c_p dT - \alpha dp = dq = -L dw_s \tag{1}$$

The heat exchange between the air parcel and its environment is zero, but heat is released within the air parcel by the phase change from water vapor to liquid water (ice). As vapor is converted to liquid water, $w = w_s$ decreases and $dq = -L dw_s$ remember $dw_s < 0$

$$c_p dT - \alpha dp \;=\; -L\left(\frac{\partial w_s}{\partial T} dT + \frac{\partial w_s}{\partial p} dp\right) \tag{2}$$

We can neglect the weak dependence of w_s on pressure, and use $-\alpha dp = -g dz$

$$\left(c_p + L\frac{\partial w_s}{\partial T}\right) dT \;=\; -g dz \tag{3}$$

$$\left(1 + \frac{L}{c_p}\frac{\partial w_s}{\partial T}\right) dT \;=\; -\Gamma_d dz \tag{4}$$

$$\frac{dT}{dz} \;=\; -\Gamma_d \frac{1}{\left(1 + \frac{L}{c_p}\frac{\partial w_s}{\partial T}\right)} \tag{5}$$

$$-\frac{dT}{dz} \;=\; \Gamma_s \tag{6}$$

$$\tag{7}$$

This means that the pseudoadiabatic lapse rate (Γ_s) is not constant, but a function of altitude. Notice that the denominator is larger than unity, so the denominator exceeds the numerator so $\Gamma_s < \Gamma_d$. This means that the temperature changes more slowly along a pseudoadiabat than along a dry adiabat. The reason is the release of latent heat by the condensation of vapor to liquid water.

- Because w_s increases with increasing T, so Γ_s decreases with increasing temperature.

- The slope of a pseudoadiabat increases and approaches that of a dry adiabat as pressure and temperature decrease.

2) DERIVATION OF EQUIVALENT POTENTIAL TEMPERATURE

Analogous to the potential temperature, which is conserved along the dry adiabat, the quantity that is conserved along a pseudoadiabat is the equivalent potential temperature. We begin, again, with the First Law (Equation 1), use $p\alpha = RT$ and divide by T.

$$c_p \frac{dT}{T} - R_d \frac{dp}{p} = \frac{-Ldw_s}{T} \tag{8}$$

We can also take the $\ln$ of the potential temperature $\ln\theta = \ln T + \kappa \ln p_0 - \kappa \ln p$, differentiating and multyplying by c_p we have:

$$c_p \frac{d\theta}{\theta} = c_p \frac{dT}{T} - R_d \frac{dp}{p} \tag{9}$$

which means that

$$\frac{-L}{c_p T} dw_s = \frac{d\theta}{\theta} \tag{10}$$

we can demonstrate that $\frac{L}{c_p T} dw_s \approx d\left(\frac{Lw_s}{c_p T}\right)$, so we can integrate to obtain:

$$d\left(\frac{Lw_s}{c_p T}\right) + d\ln\theta = d\ln\left[\theta exp\left(\frac{Lw_s}{c_p T}\right)\right] = 0 \tag{11}$$

Integrated to obtain

$$\theta_e = \theta exp\left(\frac{Lw_s}{c_p T}\right) = constant \tag{12}$$

The equivalent potential temperature is conserved along a pseudoadiabat. Θ_e is the potential temperature Θ of a parcel of air when all the water vapor has condensed so that its saturation mixing ratio w_s is zero. Θ_e can be found as follows: the air is lifted pseudoadiabatically until all the vapor has condensed, released its latent heat, and fallen out. The air is then compressed dry adiabatically to the standard pressure of 1000hPa, at which point it will attain the temperature Θ_e. If the air is initially unsaturated, w_s and T are the saturation mixing ratio and temperature at the point where the air first becomes saturated after being lifted dry adiabatically.

a. Moist Static Energy

When height, rather than pressure, is used as the independent variable, the conserved quantity during adiabatic or pseudo adiabatic ascent or descent with water undergoing transitions between liquid and vapor phases is the moist static energy (MSE).

$$MSE = c_p T + \Phi + Lq \tag{13}$$

where T is the temperature of the parcel, Phi is the geopotential and q is the specific humidity. The first term on the right side is the enthalpy per unit mass f air. The second term is the potential energy, and the third term is the latent heat content. The first two terms are the *dry static energy*. When air is lifted dry adiabatically, enthalpy is converted into potential energy and the latent heat content remains unchanged. In saturated adiabatic ascent, energy is exchanged among all three terms - potential energy increases, while the enthalpy and latent heat content both decrease. However, the sum of the three terms remains constant.

1) Adiabatic Wet Bulb Temperature T_{aw}

Point (T_{aw}, p) on the $\theta_e(T_s, p_s)$ pseudoadiabat which, is given by:

$$T_{aw}\left(\frac{1000}{p}\right)^{\kappa} exp\left(\frac{Lw_s(T_{aw}, p)}{c_p T_{aw}}\right) = \theta_e(T_s, p_s) \tag{14}$$

On a skew T, it is given by the isotherm passing through the intersection of the $\theta_e(T_s, p_s)$ pseudoadiabat with the isobar p.

2) Adiabatic Wet-Bulb Potential Temperature θ_{aw}

Point $(\theta_{aw}, 1000)$ on the $\theta_e(T_s, p_s)$ pseudoadiabat given by

$$\theta_{aw} exp\left(\frac{Lw_s(\theta_{aw}, 1000)}{c_p \theta_{aw}}\right) = \theta_e(T_s, p_s) \tag{15}$$

3) Adiabatic Equivalent Temperature T_{ae}

If we follow the pseudoadiabat $(\theta_e(T_s, p_s))$ to the top of the atmosphere, all the water is condensed out and $r = r_s = 0$. Because the air is completely dry, the pseudoadiabat is equal to the dry adiabat given by:

$$T_{ae}\left(\frac{1000}{p}\right)^{\kappa} = \theta_e(T_s, p_s) \tag{16}$$

$$T_{ae}\left(\frac{p}{1000}\right)^{\kappa} \theta_e(T_s, p_s) \tag{17}$$

On a SkewT, follow the pseudoadiabat up to the lowest pressure, then down to the dry adiabat until it intersects the p isobar.

4) Adiabatic Equivalent Potential Temperature θ_{ae}

The temperature that crosses the dry adiabat passing through T_{ae} at p=1000.

$$\theta_{ae} = \theta_e(T_s, p_s) \tag{18}$$

Obtained by following the pseudoadiabat to the lowest pressure on the chart and down the dry adiabat through this point to the 1000 mb isobar. This is the way to determine the value of $\theta_e(T_s, p_s)$ on a skew T, log p graph.