NCAR (National Center for Atmospheric Research) has an excellent resource for education called COMET-MetEd. There you can find some really great tutorials on SkewT-LogP plots: visit http://www.meted.ucar.edu/mesoprim/skewt/index.htm. You have to register, but it is free to use for anyone.

1. Static Stability

1a. Static Stability of Unsaturated Air

If an air parcel of volume $V$ and density $\rho'$ is displaced from its initial position $(z, p)$ where there is no net force on it, to $(z + z', p - \Delta p)$. From Newton’s Second Law of Motion $F = ma$ we have

$$ \left(\rho'V\right) \frac{d^2z'}{dt^2} = g\rho(V) - g\rho'(V) $$

(1)

$$ \frac{d^2z'}{dt^2} = g \left( \frac{\rho - \rho'}{\rho'} \right) $$

(2)

$$ = g \left( \frac{1/T - 1/T'}{1/T'} \right) $$

(3)

$$ = g \left( \frac{T' - T}{T} \right) $$

(4)

If $T_0$ is the environmental temperature at $z$, the new temperature of the air parcel at $z + z'$ will be $T' = T_0 - \Gamma_d z'$, and the ambient temperature will be $T = T_0 - \Gamma z'$, so $T' - T = -(\Gamma_d - \Gamma)z'$.

So the equation of motion can be written:

$$ \frac{d^2z'}{dt^2} = -\frac{g}{T}(\Gamma_d - \Gamma)z' $$

(5)

$$ \frac{d^2z'}{dt^2} + N^2z' = 0 $$

(6)

Where $N = \left[ \frac{g}{T}(\Gamma_d - \Gamma) \right]^{1/2}$ is the Brunt-Vaisala frequency. Equation 6 is a second order ordinary differential equation.
Remember: A second order ordinary differential equation of the form \( ay'' + by' + c = 0 \) has a solution:

<table>
<thead>
<tr>
<th>Roots of ( ar^2 + br + c = 0 )</th>
<th>General solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_1 ) and ( r_2 ) real and distinct ( r_1 = r_2 = r )</td>
<td>( y = c_1 e^{rx} + c_2 e^{r_2 x} )</td>
</tr>
<tr>
<td>( r_1, r_2 ) complex ( \alpha \pm i\beta )</td>
<td>( y = c_1 e^{rx} x + c_2 e^{r_2 x} x )</td>
</tr>
</tbody>
</table>

The three possible cases are:

\( N^2 > 0 \) if \( \Gamma < \Gamma_d, \frac{d\theta}{dz} > 0 \)

\( N^2 < 0 \) if \( \Gamma > \Gamma_d, \frac{d\theta}{dz} < 0 \)

\( N^2 = 0 \) if \( \Gamma = \Gamma_d, \frac{d\theta}{dz} = 0 \)

i. **Stable (subadiabatic)** \( N^2 > 0, \Gamma < \Gamma_d \). Environmental temperature decreases with height slower than the displaced parcel’s temperature. The parcel experiences a buoyancy force that opposes the displacement \( z' \). If it is displaced upward, the parcel becomes heavier than its surroundings and thus negatively buoyant. This buoyancy reactions constitutes a positive restoring force. The layer is said to be hydrostatically stable.

In this case, the solution to Equation 6 has a solution of the type:

\[
z'(t) = c_1 e^{iNt} + c_2 e^{-iNt}
\]  

\( c_1 \) and \( c_2 \) are determined by the initial conditions. The solution is the *Brunt-Vaisala oscillation* at the frequency \( N \) and period \( \tau_b = 2\pi/N \)

ii. **Neutral Stability** \( N^2 = 0, \Gamma = \Gamma_d \). The environmental lapse rate decreases with height at the same rate as the parcel’s temperature. The buoyancy force vanishes and the displaced parcel experiences no restoring force. The atmospheric layer is said to be hydrostatically neutral. In this case, the solution to Equation 6 has a solution of the type:
\[ z'(t) = z'(t = 0) + \left( \frac{dz'}{dt} \right)_{t=0} t \] (8)

(Using the initial conditions). Consequently, because the density of the air parcel and ambient atmosphere are the same, the buoyancy force and weight are equal and opposite and there is not net force on the air parcel.

iii. Unstable (subadiabatic) \( N^2 < 0 \) \( \Gamma > \Gamma_d \). Environmental temperature decreases with height faster than the parcel’s temperature. The parcel experiences a buoyancy force that reinforces the displacement \( z' \). If it is displaced upward, the parcel becomes lighter than its surroundings and thus positively buoyant. This buoyancy reaction constitutes a negative restoring force, one that drives the system away from its undisturbed elevation - irrespective of the sense of the displacement. The atmospheric layer is then said to be hydrostatically unstable. In this case, the solution to Equation 6 has a solution of the type:

\[ z'(t) = c_1 e^{-|N|t} + c_2 e^{|N|t} \] (9)

\( c_1 \) and \( c_2 \) are determined by the initial conditions. The parcel continues moving in the direction in which it was displaced. When an environmental layer is unstable, dry convection occurs with warm air parcels rising from the bottom of the layer and cold air parcels sinking from the top.

2. Stability in Terms of Potential Temperature

Stability criteria can also be expressed in terms of potential temperature. For unsaturated conditions,

\[ c_p T \frac{d\theta}{\theta} = c_p dT + g dz \] (10)

Letting \( d\theta = (\partial \theta / \partial z) dz \) and \( dT = (\partial T / \partial z) dz \) and dividing through by \( c_p T dz \) yields

\[ \frac{1}{\theta} \frac{\partial \theta}{\partial z} = \frac{1}{T} \left[ \frac{\partial T}{\partial z} + \frac{g}{c_p} \right] = \frac{1}{T} \left[ \Gamma_d - \Gamma \right] \] (11)

A layer within the troposphere in which \( \Gamma < 0 \) or \( dT/dz > 0 \) is called an
Table 1: Summary for Unsaturated Air

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Lapse Rate</th>
<th>$d\theta/dz$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stable</td>
<td>$\Gamma &lt; \Gamma_d$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>Neutral</td>
<td>$\Gamma = \Gamma_d$</td>
<td>$= 0$</td>
</tr>
<tr>
<td>Unstable</td>
<td>$\Gamma &gt; \Gamma_d$</td>
<td>$&lt; 0$</td>
</tr>
</tbody>
</table>

inversion layer where the air is stably stratified. If the atmospheric boundary layer is capped by an inversion layer, the air within the boundary layer cannot penetrate vertically across the inversion layer.

3. Stability of Saturated Air

As a saturated air parcel is displaced upwards, its temperature will decrease at the pseudoadiabatic lapse rate. If the ambient lapse rate is greater than the pseudoadiabatic lapse rate, $\Gamma > \Gamma_s$, the displaced parcel will be warmer than its environment and will be accelerated in the direction of the displacement. This air is unstable with respect to the pseudoadiabatic parcel displacement. If we allow for condensation on ascent, five regimes can exist:

Table 2: Summary for Unsaturated Air

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Lapse Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolutely Stable</td>
<td>$\Gamma &lt; \Gamma_s$</td>
</tr>
<tr>
<td>Neutral (saturated)</td>
<td>$\Gamma = \Gamma_s$</td>
</tr>
<tr>
<td>Conditionally Unstable</td>
<td>$\Gamma_d &gt; \Gamma &gt; \Gamma_s$</td>
</tr>
<tr>
<td>Neutral (unsaturated)</td>
<td>$\Gamma = \Gamma_d$</td>
</tr>
<tr>
<td>Absolutely Unstable</td>
<td>$\Gamma &gt; \Gamma_d$</td>
</tr>
</tbody>
</table>

4. Latent Instability

We now consider the stability conditions in a conditionally unstable layer, such as that extending from the surface, point P1 to the point R of the sounding shown in
the figure. When an air parcel rises vertically in the atmosphere, a certain amount of work per unit mass is performed either by the buoyancy force against gravity or vice versa. The work is given by:

\[ w = \frac{1}{m} \int_a^b \mathbf{F} \cdot \mathbf{ds} = \frac{1}{m} \int_a^b F \, dz = \int_a^b \frac{d^2 z'}{dt^2} \, dz \]  

(12)

with \( \frac{d^2 z'}{dt^2} = g \left( \frac{T' - T}{T} \right) \).

\[ w = g \int_a^b \frac{T' - T}{T} \, dz \, dp = R \int_a^b (T' - T) \, d(-\ln p) \]  

(13)

\[ w = R(A' - A) \]  

(14)

for \( A' = \int_a^b T' \, d(-\ln p) \) and \( A = \int_a^b T \, d(-\ln p) \).
Table 3: Emagram (Energy per unit mass diagram)

A diagram of $T$ vs $\ln p$ is a true thermodynamic diagram on which the area is proportional to energy (or work). The area enclosed in any contour is proportional to the work done in a cyclic process defined by the contour.

\[
dw = p\,d\alpha = RdT - \alpha dp = RdT - RT\,d\ln p
\]
\[
\oint dw = \oint RdT - \oint RT\,d\ln p = -\oint RT\,d\ln p
\]
The most elementary approach to find the vertical velocity of a convective element is:
The work per unit mass will be transformed into kinetic energy:

\[
\begin{align*}
w &= \int_a^b \frac{d^2 z}{dt^2} \, dz = \int_a^b \frac{dv}{dt} \, dz \quad = \quad \int_a^b \frac{dv}{dt} \, dt = \frac{1}{2} \int_a^b dv^2 = \frac{1}{2} (v_b^2 - v_a^2) \\
\text{so} \\
\frac{1}{2} (v_b^2 - v_a^2) &= R (A' - A)
\end{align*}
\]

When the process curve \( C' \) lies to the right of the sounding curve \( C \), the buoyancy force exceeds the weight of the air parcel, hence \( A' > A \), \( w > 0 \) and \( v_b^2 > v_a^2 \). In this case, there is upward acceleration. If the process curve lies to the left of the sounding curve \( C \), work equal to the negative area on the emagram must be done on the air parcel to cause it to ascend.

In this case, energy (work) equal to the positive area on the emagram is available to accelerate an air parcel upward from \( a \) to \( b \) along \( C' \). When the process curve \( C' \) lies to the left of the sounding curve \( C \), the weight of the air parcel exceeds the buoyancy force, hence \( A' < A \), \( w < 0 \) and \( v_b^2 < v_a^2 \). In this case work equal to the negative area on the emagram must be done on an air parcel to cause it to ascend from \( a \) to \( b \) along \( C' \).

In order for an air parcel to follow path \( C1' \), work proportional to area \( A - \) must be performed (for example through orographic ascent). The level at which the air parcel reaches saturation, point \( Ps \) is the lifting condensation level (LCL). When the air parcel surpasses the level at which the process curve \( C1' \) crosses the sounding, at LFC, the net buoyancy force performs work proportional to area \( A+ \). Convection will thus continue freely until \( C1' \) crosses \( C \) again at the equilibrium level (EL), and the parcel decelerates. The EL is the height where the temperature of a buoyantly rising parcel again equals the temperature of the environment. The level of the first crossing, LFC, is called the level of free convection (LFC). The total area \( A+ \) measures the latent instability for an air parcel at level \( P1 \). We can proceed similarly with parcels from the other levels above \( P1 \). When the level of \( P2 \) is reached, \( A+ \) vanishes. This is then the upper limit of the layer for which
there is latent instability.

The convective available potential energy (CAPE) is represented by the area on a skw-T diagram enclosed by the environmental temperature profile and the moist adiabatic funning from the LFC to the EL. This area indicates the amount of buoyant energy available as the parcel is accelerated upward. CAPE is measured in units of joules per kilogram (J/kg). The larger the positive area, the higher the CAPE value and the instability, and the greater the potential for strong and perhaps severe convection. This table offers a general correlation between CAPE and atmospheric stability, however CAPE climatologies vary widely.

\[
CAPE = R \int_{p_{LFC}}^{p_e} (T' - T)(-\ln p) \, dp
\]

Table 4: CAPE

<table>
<thead>
<tr>
<th>CAPE Value</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Stable</td>
</tr>
<tr>
<td>0-1000</td>
<td>Marginally unstable</td>
</tr>
<tr>
<td>1000-2500</td>
<td>Moderately Unstable</td>
</tr>
<tr>
<td>2500 - 3500</td>
<td>Very Unstable</td>
</tr>
<tr>
<td>3500 or greater</td>
<td>Extremely Unstable</td>
</tr>
</tbody>
</table>

CAPE may also be related to updraft velocity via the relation \(w_{max} = (2 \times CAPE)^{1/2}\). CAPE is computed automatically and displayed as output in electronic versions of the skew-T diagram. When calculating CAPE, we normally lift a parcel that reflects the mean values of the temperature and moisture in the lowest 50-100mb. This layer represents the average heat and moisture conditions fueling convective storms.

While CAPE is a robust indicator for deep convection and convective stability, the computation of CAPE is extremely sensitive to the mean mixing ratio in the lowest 500m. A 1g/kg increase can increase cape by 20%. Furthermore, CAPE doesn’t account for wind shear, so it may underestimate the potential for severe convection where strong shear is present.

**Convective Inhibition (CIN)** is represented by the area on a skew-T diagram enclosed by the environmental temperature profile and the temperature of a
parcel lifted from some originating level to the LFC. This area indicates the amount of energy required to lift the parcel to the LFC. CIN is measured in units of joules per kilogram. The larger the negative area, the higher the CIN value and the lower the likelihood of convective storms. One caveat is that if the CIN is large but storms manage to form, usually due to increased moisture and/or heating, then the storms are more likely to be severe.

4a. *Changing Stability*

*Instability from surface heating:* The ground absorbs solar radiation, causing the surface temperature to rise. This in turn heats the surface air parcels by conduction. These heated parcels tend to organize in the large "bubbles" that ascend due to buoyancy. If the lapse rate is adiabatic or superadiabatic the bubbles rise rapidly until a stable region is reached which resists further rising motions. On the other hand if the lapse rate is stable then the rise of the parcels is delayed. Bubbles will start to rise wither when some of them become sufficiently warm to spontaneously rise or when some are impelled upward by mechanical turbulence. The rising parcels will then penetrate some distance into the overlying stable region. Through such penetrative convection, heat bubbles warm the lowest atmosphere. With time, warm air rises to higher and higher altitudes in the stable layers above, thereby slowly extending a dry-adiabatic lapse rate from the surface to greater altitudes. Thus surface heating creates instability indirectly through the intermediate mechanism of convective mixing. (see COMET website on Changing Stability). The lowest part of the sounding becomes substituted by dry adiabats, and the top of the layer can reach saturation, this is called the Convective Condensation Level (CCL). From that moment on, convection may proceed spontaneously along the dry adiabat without any need for forced lifting. Conversely, pure nocturnal radiative cooling in calm air results in a shallow surface-based inversion. The depth of the inversion increases with greater duration of the cooling, while the strength of the inversion increases with the degree of cooling. Evaporative cooling of precipitation can also produce significant cooling in an unsaturated boundary layer resulting in a more stable environment.

**Convective Condensation Level (CCL)** is the height to which a parcel of air, if heated sufficiently from below, will rise adiabatically until it is just saturated. Usually it is the height of the base of cumuliform clouds produced by thermal convection caused solely by surface heating. To determine the CCL
on a sounding, start at the surface dew point, proceed upward along the saturation mixing ratio line until this line intersects the temperature profile on the sounding. The level of the intersection is the CCL.

**Convective Temperature** $T_c$ is the surface temperature that must be reached to start the formation of convective clouds caused by solar heating of the near-surface layer. From the CCL, proceed downward along the dry adiabatic to the surface pressure isobar.

*Moistening* Increasing the surface dew point can also increase the instability as it lowers the LCL so parcels don’t have to lifted as far to become unstable. In other words, segment $P_1P_s$ becomes shorter, $A-$ decreases and $A+$ increases.

In practice, the vertical velocity and penetration height of a cumulus tower are reduced from adiabatic values by nonconservative effects. Cooler and drier environmental air that is entrained into and mixed with a moist thermal depletes ascending parcels of positive buoyancy and kinetic energy.

### 5. Potential or Convective Instability

We have so far considered the stability properties of the atmosphere when an isolated mass, an air parcel, is vertically displaced. This occurs, for example, when the warming of the lower layers causes the ascent of air masses with dimensions of the order of hundreds of meters to 1km. These masses can eventually become accelerated by latent instability. They are called thermals or bubbles, and become visible as cumulus convective clouds.

It is also important to study the vertical movements of an extended layer of air, such as may occur for example during forced ascent of an air mass over an orographic obstacle, or due to large-scale ascent during fronts. We now consider this case and derive the effect of the movement on the lapse rate of the layer.

#### 5a. Layer is and Remains Unsaturated

We consider a layer of thickness $\delta z$ which ascends from pressure $p$ to $p'$, where it has a thickness $\delta z'$ (unprimed quantities denote the lower location of the layer, primed quantitates the upper location. During ascent, which we take to be adiabatic, the virtual potential temperatures of the bottom and top of the layer, $\theta_v$ and $\theta_v + \delta \theta_v$ do not change, so the variation of $\delta \theta_v$ through the layer also remains constant. The rate of change of $\theta_v$ with increasing altitude at the base of the layer must then be inversely proportional to $\delta z$. 

10
\[ \frac{\delta \theta_v}{\delta z} = \left( \frac{\delta \theta_v}{\delta z'} \right) \frac{\delta z'}{\delta z} \] (18)

Conservation of mass requires \( \rho A \delta z = \rho' A' \delta z' \), so

\[ \frac{\delta z'}{\delta z} = \frac{pT' v_A}{p'T_v A'} \] (19)

Combining the two above equations:

\[ \frac{\delta \theta_v}{\delta z} = \frac{pT' v_A}{p'T_v A'} \left( \frac{\delta \theta_v}{\delta z'} \right)' \] (20)

We have already demonstrated that \( \frac{\delta \theta_v}{\delta z} = \frac{\theta}{T_v} (\Gamma_d - \Gamma) \) through Equation 11, similarly \( \left( \frac{\delta \theta_v}{\delta z} \right)' = \frac{\theta}{T_v'} (\Gamma'_d - \Gamma') \).

So we find that:

\[ \Gamma' = \Gamma + (\Gamma_d - \Gamma) \left[ 1 - \frac{p'A'}{pA} \right] \] (21)

\[ \Gamma = \Gamma' + (\Gamma_d - \Gamma') \left[ 1 - \frac{pA}{p'A'} \right] \] (22)

We can have the following situations:

\( \Gamma = \Gamma_d \) In this case, the layer lapse rate is dry adiabatic and the ascent occurs along the same adiabatic. This means that the ascent or descent of a neutral layer does not change the layer’s neutral stability.

\( \Gamma < \Gamma_d \) For ascent, both the vertical stretching, \( A' < A \) and pressure decrease, \( p' < p \), increase the lapse rate \( \Gamma' > \Gamma \). Thus ascent of a stable layer decreases its stability. As the ascent increases \( p'A'/pA \) tends to zero and \( \Gamma' \) tends to \( \Gamma \). This can occur in cyclones where the convergence aloft may lead to nearly adiabatic lapse rates.

For descent, vertical shrinking and pressure increase decrease the lapse rate \( \Gamma < \Gamma' \). Thus the descent of a stable layer increases its stability. If the descent is sufficiently large, the sign of the lapse rate can change, resulting in inversions. This can occur in anticyclones where the subsidence frequently gives rise to inversions.

\( \Gamma > \Gamma_d \) Doesn’t occur because it corresponds to absolute instability.
Figure 1: Layer remains unsaturated during ascent
5b. Part of the Layer Becomes Saturated During the Ascent

If a layer is more humid at its base than at its top (see left Figure 2) $\delta \theta_{aw}/\delta z < 0$), the base will become saturated before the top in rising motion. From that moment the base of the layer will follow the saturated adiabatic, while the top will follow the dry adiabatic. Thus, the ascent makes the layer less stable and can make it absolutely unstable. This is expressed by saying that the layer was originally potentially unstable. This layer will tent to produce cumuloform clouds.

If a layer is more humid at its top than at its base (see right Figure 2) $\delta \theta_{aw}/\delta z > 0$), the top will become saturated before the base in rising motion. From that moment the top of the layer will follow the saturated adiabatic while the base follows the dry adiabatic. Thus the ascent makes the layer more stable. This is expressed by saying that the layer was originally potentially stable. This layer tends to produce stratiform clouds.
Figure 2: (Left) Potentially unstable layer becoming partly saturated $\delta \theta_{aw}/\delta z < 0$, (Right) Potentially stable layer becoming partly saturated $\delta \theta_{aw}/\delta z > 0$