A fundamental limit on electric fields in air

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[1] By modifying the avalanche mode of runaway breakdown to include positive feedback from gamma-rays and positrons, it is found that enormous bursts of energetic radiation can be produced in strong electric fields in air, with peak fluxes up to one billion times greater than from conventional models. These bursts generate so many runaway electrons that the electric field is very rapidly discharged, resulting in a fundamental upper limit on the electric field strength achievable in air. This limit has important implications for the electrification of thunderstorms and the production of lightning. *INDEX TERMS:* 3304 Meteorology and Atmospheric Dynamics: Atmospheric electricity; 3300 Meteorology and Atmospheric Dynamics; 3324 Meteorology and Atmospheric Dynamics: Lightning; 3329 Meteorology and Atmospheric Dynamics: Mesoscale meteorology; 3367 Meteorology and Atmospheric Dynamics: Theoretical modeling. **Citation:** Dwyer, J. R., A fundamental limit on electric fields in air, *Geophys. Res. Lett.*, *30*(20), 2055, doi:10.1029/2003GL017781, 2003.

1. Introduction

[2] Two longstanding problems in the physics of thunderstorms and lightning are determining the electric fields necessary for the initiation and propagation of lightning, and understanding the maximum electric fields found inside thunderstorms [*Phelps and Griffiths*, 1976; *MacGorman and Rust*, 1998; *Solomon et al.*, 2001]. In situ measured electric fields are rarely large enough to produce conventional breakdown [*Winn et al.*, 1974; *Marshall and Rust*, 1991; *Marshall et al.*, 1995], leading some investigators to suggest that high electric fields might actually exist, but only in localized, unmeasured regions. Alternatively, high fields might also occur for durations shorter than the integration times of electric field instruments and for this reason have not been observed. Because detailed mappings of the electric fields inside thunderstorms is not practical, it is difficult to determine what conditions are necessary for the initiation and subsequent propagation of lightning discharges. In this paper, a new fundamental upper limit on the electric field strength in air is presented, thereby answering the more basic question: What is the strongest electric field attainable? This upper limit, which also applies to clear, dry air, is much smaller than the conventional electric breakdown field strength and therefore places a very strict bound on the electric fields that are even possible in thunderstorms. Indeed, the mechanism proposed here may in some cases explain the maximum electric fields measured inside storms.

2. Runaway Breakdown of Air

[3] One class of models for describing discharges in thunderstorms incorporates the runaway breakdown of air, which has an electric field threshold about an order of magnitude lower than that needed for a conventional breakdown [*Wilson*, 1925; *Gurevich et al.*, 1992; *Gurevich and Zybin*, 2001]. Runaway electrons are produced in air when the energy gained from the electric field exceeds the losses from collisions, allowing the electrons to accelerate to relativistic energies. An avalanche of such runaway electrons develops when energetic knock-off electrons are produced via hard elastic (Møller) scattering with electrons in the air molecules. These knock-off electrons subsequently run away, producing more energetic knock-off electrons and so on. The runaway electrons in the avalanche produce large quantities of ionization plus x-rays and gamma-rays through bremsstrahlung interactions with air [*Gurevich et al.*, 1997]. In addition, the high-energy gamma-rays generate a smaller number of positrons through pair production, some of which can also run away, but in the opposite direction of the electrons [*Gurevich et al.*, 2000]. The existence of runaway breakdown seems likely given recent observations of energetic radiation associated with natural and triggered lightning, thunderstorms and red-sprites [*Moore et al.*, 2001; *Dwyer et al.*, 2003; *Parks et al.*, 1981; *Eack et al.*, 1996; *Fishman et al.*, 1994].

3. Monte Carlo Simulation

[4] In this paper, results are presented from a new 3-D Monte Carlo simulation of the runaway breakdown of air. This simulation builds upon earlier work by *Lehtinen et al.* [1999] and includes, in an accurate form, all the important interactions involving runaway electrons, including energy losses through ionization and atomic excitation and Møller scattering. Unlike most earlier work, however, this simulation fully models elastic scattering using a shielded-Coulomb potential, rather than relying on a diffusion approximation, and also includes bremsstrahlung production of x-rays and gamma-rays and the subsequent propagation of the photons, including photoelectric absorption, Compton scattering and pair production. In addition, new features include the incorporation of positron propagation and the generation of energetic seed electrons via Bhabha scattering of positrons and via Compton scattering and photoelectric absorption of energetic photons. In this paper, the effects of positive feedback mechanisms involving the positrons and energetic photons are presented. Such positron and gamma-ray feedback occurs when positrons and

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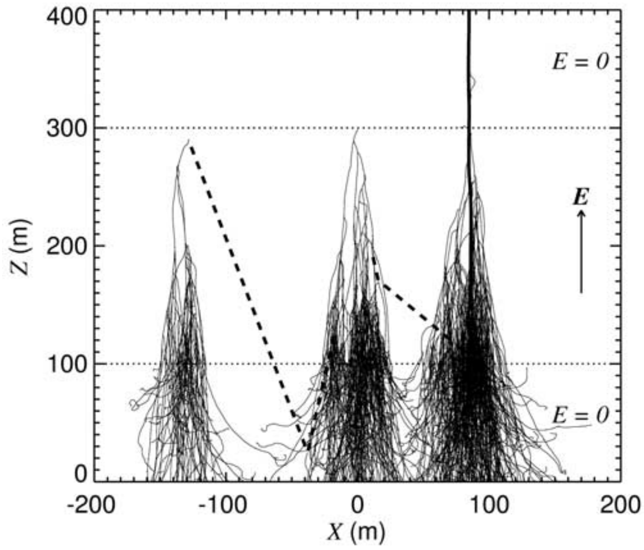

Figure 1. Partial results of the Monte Carlo simulation showing the runaway breakdown of air. The light tracks are the runaway electrons, the dashed lines are the gamma-rays and the dark track is a positron. The entire avalanche is initiated by one, 1 MeV, seed electron injected at the top center of the volume. The horizontal dotted lines show the boundaries of the electric field volume (E = 1000 kV/m). For clarity, only a small fraction of the runaway electrons and gamma-rays produced by the avalanche are plotted. The avalanches on the left and right illustrate the gamma-ray feedback and positron feedback mechanisms, respectively.

gamma-rays, produced by the runaway avalanche, propagate to the electric field region with the highest negative potential and produce more energetic seed electrons.

[5] Figure 1 shows partial results of the Monte Carlo simulation, illustrating the processes involved in the runaway breakdown of air. In the figure, one high-energy (1 MeV) seed electron is injected at the top center of the region containing a uniform electric field. This electron runs away, producing an avalanche of relativistic electrons (light tracks). Bremsstrahlung gamma-rays (dashed lines) are produced when the runaway electrons collide with air. The figure also illustrates the two feedback mechanisms that generate additional seed electrons, producing more runaway breakdown. *Positron feedback:* The gamma-ray on the right side of the figure produces a positron (dark trajectory on right) via pair production. This positron runs away, traveling to the top of the figure and producing more runaway electrons via hard elastic scattering, resulting in the secondary avalanche on the right. Because the positron is quickly accelerated to relativistic energies, it can travel many hundreds of meters before annihilating. *Gamma-ray feedback*: The gamma-ray on the left side of the figure Compton scatters to the top and produces another seed electron via the photoelectric effect (shown) or via Compton scattering. This seed electron then runs away producing the secondary avalanche on the left. These secondary avalanches, in turn, produce more feedback electrons via the two mechanisms described above, allowing the whole process to increase exponentially. This mechanism is analogous to the Townsend discharge, which describes conventional electric breakdown caused by feedback mechanisms involving positive ions and optical photons [*Brown*, 1966]. However, unlike the Townsend discharge, which involves only low-energy particles (<100 eV), the present mechanism involves high-energy particles with energies up to many millions of electron volts.

[6] In order to study the role of positron and gamma-ray feedback in runaway breakdown, the simulation uses a uniform electric field, with magnitude greater than the runaway threshold, directed up along the axis of a cylindrical volume of length L and radius R with $R \gg L$. Outside the volume, the field is zero, and all particles continue to be propagated after they leave the volume until their energy is lost. The generally small effects caused by the earth's magnetic field are ignored, and variations in atmospheric pressure are not included. Each simulation is initiated by injecting a high-energy seed electron into the top of the simulation volume, representing, for example, a knock-off electron produced by a cosmic-ray muon. The avalanche is then propagated until all the particles leave the volume and lose their energy.

[7] A useful parameter for describing runaway breakdown is the characteristic length for an avalanche to develop, λ [*Gurevich and Zybin*, 2001]. The number of runaway electrons in one avalanche, produced by N_o seed electrons, is given by $N_{re} = N_o \exp(z/\lambda)$, where z is the distance from the start of the avalanche. The dependence of λ on the electric field strength as calculated by the simulation is shown in Figure 2. For 300 kV/m $\leq E \leq$ 2500 kV/m, λ is well fit by the empirical formula

$$\lambda = 7200kV \times (E - 275kV/m)^{-1}, \tag{1}$$

where E is the electric field strength measured in kV/m. (Note: throughout this paper all results are for a pressure of

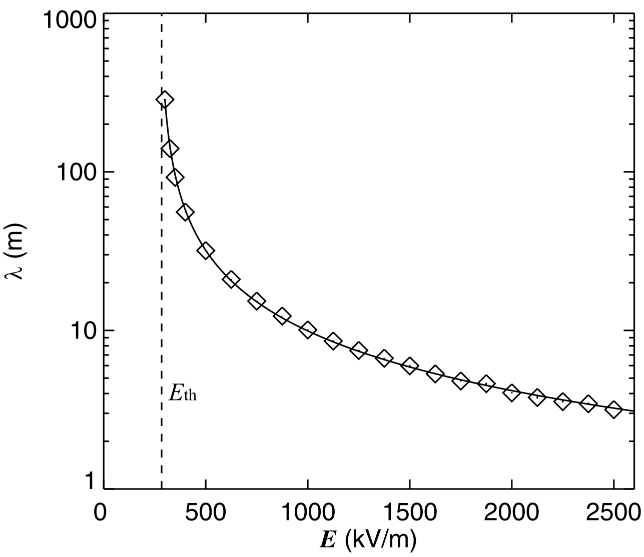

Figure 2. Characteristic length for an avalanche to develop, λ, as a function of the electric field strength, E. The data points are calculated by the Monte Carlo simulation and the solid curve is given by Equation 1. The vertical dashed line shows the threshold, E_{th}, for runaway breakdown to occur.

1 atm. For lower pressures, such as is applicable at high altitudes, all electric field strengths are simply reduced by a factor of P and all length scales are increased by a factor of P^{-1}, where P is the atmospheric pressure in units of atm.) The actual threshold for runaway breakdown, E_{th}, is found by the simulation to be 284 kV/m (vertical dashed line). This threshold is 25% higher than the values widely discussed in the literature. The difference is likely due to differences in the elastic scattering calculation, which is very important for fields close to E_{th}. As a test of the simulation, when bremsstrahlung production is temporarily suppressed, the resulting avalanche rates calculated by the simulation agree within 10% with published rates by *Lehtinen et al.* [1999].

[8] To keep track of the production of feedback particles, the cylindrical volume is divided into two smaller cylinders of height $L/2$. The number of feedback electrons produced in the upper volume per electron that originally enters the lower volume is defined to be the amplification factor, γ, in direct analogy with the 2nd Townsend coefficient for conventional discharges [*Brown*, 1966]. The growth of the runaway electrons, initiated by N_o seed electrons at the top of the volume, can then be described approximately by

$$N_{re} = N_o \gamma^{t/\tau} \exp(L/\lambda), \tag{2}$$

where N_{re} is the number of runaway electrons and τ is the average time required to complete one cycle, from the time the initial electron enters the lower volume to the time the next batch of feedback electrons enter it. For $E > 350$ kV/m, τ is found to be less than 10 μsec, and for $E > 500$ kV/m, τ is less than 3 μsec. Without the feedback mechanism, the entire breakdown process would stop when the runaway electrons in the initial avalanche reach the bottom of the volume, producing only $N_{re} = N_o\exp(L/\lambda)$ runaway electrons. Of course, over time as more cosmic-rays produce additional seed electrons, the overall number of runaway electrons will increase. However, this number only increases linearly with time, not exponentially as when feedback is included. When feedback is included and when $\gamma > 1.0$, the number of energetic electrons increases exponentially with time until eventually so much secondary ionization is produced that the polarization of the electron-ion cloud reduces the electric field until γ falls below 1.

[9] Figure 3 shows the electric field, E_{max}, necessary to make $\gamma = 1.0$ (i.e., the condition necessary for self-sustained runaway breakdown), as a function of the length of the electric field region, L. For electric fields >500 kV/m ($L < 340$ m), the gamma-ray feedback mechanism dominates, and for lower fields and longer distances, positron feedback is most important. The plateau at 2550 kV/m corresponds to the conventional electric breakdown field, which occurs via the streamer breakdown mechanism [*Raether*, 1964]. Without the runaway breakdown mechanism described in this paper, 2550 kV/m would be the upper limit on the allowed electric field for clear, dry air. In almost all practical cases, E_{max}, shown in Figure 3, represents the actual maximum static field obtainable in air, since the simulation shows that $\gamma \propto \exp(L/\lambda(E))$, where $\lambda(E)$ is given by equation (1). If E or L were increased above the E_{max} curve, the amplification factor would rise very rapidly and a large burst of runaway electrons would be produced, shorting out the field. Since the new values of E_{max} are independent of such quantities as the humidity, the details of the cosmic-ray background, or the presence or absence of hydrometeors such as rain, ice particles etc., this new upper limit is fundamental to our atmosphere.

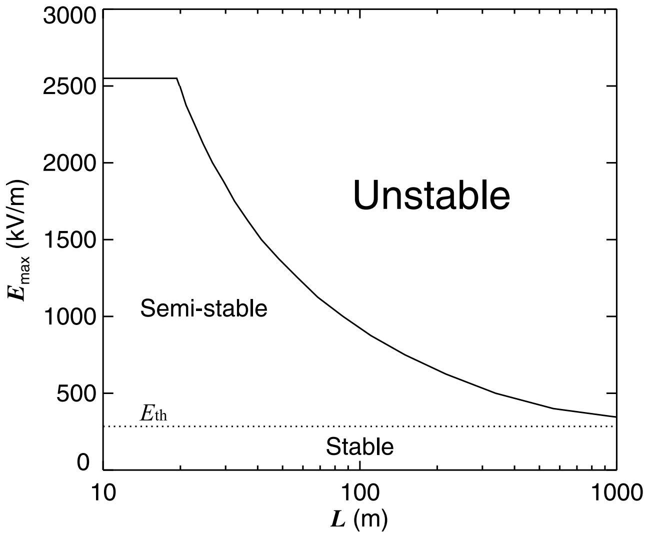

Figure 3. The maximum static electric field strength achievable in air versus the length of the electric field region at 1 atm. The horizontal dotted line shows the value of the runaway breakdown threshold, E_{th}. Above the solid curve, no electric field configuration can be maintained, and therefore the electric field is unstable. Indeed, for configurations in the upper right corner, the electric field is violently unstable. Below the curve and above E_{th}, the field may eventually discharge depending upon the ambient cosmic-ray flux and the rate of electrification. Below E_{th}, the electric field is stable when the conductivity of air is negligible.

4. Discussion

[10] While several authors have noted that runaway breakdown may discharge the large-scale electric fields inside thunderstorms [*Solomon et al.*, 2001; *Marshall et al.*, 1995; *Gurevich et al.*, 1992; *Gurevich et al.*, 1997], without the feedback mechanisms described here, this discharge can take many seconds and depends strongly upon the local cosmic-ray flux. Because electrification can, in principle, occur on a similar timescale, the runaway breakdown threshold, E_{th}, cannot be viewed as a fundamental limit on the electric field strength. Indeed, many authors have reported occasionally measuring electric field strengths well above E_{th} inside thunderstorms [*Winn et al.*, 1974; *Marshall et al.*, 1995], showing that E_{th} is not a true upper limit on E.

[11] The largest effect that can change the E_{max} curve comes from the radius of the electric field region, R. Because the gamma-rays are scattered over hundreds of meters, if the lateral width of the region is less than this amount the efficiency for producing feedback electrons is reduced. As an example, if the radius of the electric field region is set equal to only twice the length, L, then there is almost no change in the E_{max} curve for $E < 1000$ kV/m, but

the curve's value is shifted to the right by ~30% near E = 2500 kV/m.

[12] The total amount of energetic radiation produced by this runaway breakdown mechanism depends upon how quickly the electric field is reduced, which in turn depends upon the production of the space charges and the ensuing polarization of the charge cloud [*Gurevich and Zybin*, 2001]. This polarization is determined locally by the amount of ionization produced by the runaway electrons per unit length times the average length for O^- production by collision of the free, low-energy electrons with oxygen molecules. This average length is dependent upon the electric field strength but is approximately 0.01 m for the range of interest here [*Chanin et al.*, 1962]. Approximately 10^5 secondary, low-energy electrons are produced by a relativistic runaway electron per meter of air traversed. This sets a limit of approximately a few times 10^{10} runaway electrons per m^2 before the electric field is appreciably reduced. Because most of the runaway electrons are produced within one λ from the bottom, it is initially only this volume that will be discharged.

[13] Due to scattering, the lateral extent of the runaway breakdown can be large, only limited by the size of the high field region. Consequently, the total number of runaway electrons that can be produced by the entire volume is enormous. For example, for a volume with radius R = 100 m, up to $\sim 6 \times 10^{14}$ runaway electrons can be produced. If the electric field strength, E, or the size of the electric field region, L, were increased by 10% above the curve in Figure 3, then the time needed to produce 6×10^{14} runaway electrons and discharge the electric field is estimated to be less than $\sim 2 \times 10^{-4}$ sec for 350 kV/m $\leq E \leq$ 2500 kV/m. Because the flux of cosmic-rays is typically 100–1000 s^{-1} m^{-2}, a volume with radius R = 100 m would have a seed electron injected in less than a μsec. Note that this discharge time is much shorter than the many seconds needed for the electric fields to be generated inside thunderstorms [*MacGorman and Rust*, 1998] and the flux of energetic radiation from such a discharge can briefly exceed 10^{15} sec^{-1} m^{-2}!

[14] This model may help explain the large bursts of energetic radiation recently reported by *Dwyer et al.* [2003] in association with rocket-triggered lightning. Because the electric field from lightning does not remain strong over long distances, the conventional avalanche model of runaway breakdown does not easily explain this energetic radiation. However, when gamma-ray and positron feedback are incorporated, it is possible to briefly generate peak particle fluxes up to $\sim 10^9$ times larger than from standard models of runaway breakdown. Given recent observations of energetic radiation occurring just before lightning return strokes and the large amount of secondary ionization that is likely produced, it is possible that the mechanisms described here may play a role in the production of lightning.

[15] Finally, while framed in the context of thunderstorms and lightning discharges in our atmosphere, the results presented in this paper can also be viewed as a basic plasma phenomenon with applications to any gaseous medium.

[16] **Acknowledgments.** This work was supported by the NSF CAREER grant ATM 0133773.

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