1. Saturated Adiabatic Processes

If vertical ascent continues above the LCL, condensation will occur and the latent heat of phase change will be released. This latent heating will cause the temperature to decrease more slowly with pressure above the LCL than below. This means that the lapse rate above the LCL will be smaller than the lapse rate below the LCL. To describe the behavior of the parcel we use a **Pseudoadiabatic process or irreversible saturated adiabatic process** where all the condensed water or sublimated ice falls out of the air parcel as soon as it is produced. Through any point of an aerological diagram there is one, and only one, pseudoadiabat.

1) DERIVATION OF THE EXPRESSION FOR PSEUDOADIABATIC LAPSE RATE

Our objective is to find how the temperature changes as we move up the pseudoadiabat ($\Gamma_s = -dT/dz$). We begin with the First Law of Thermodynamics:

$$c_p dT - \alpha dp = dq = -L dw_s \tag{1}$$

The heat exchange between the air parcel and its environment is zero, but heat is released within the air parcel by the phase change from water vapor to liquid water (ice). As vapor is converted to liquid water, $w = w_s$ decreases and $dq = -L dw_s$ remember $dw_s < 0$

$$\begin{aligned}
c_p dT - \alpha dp &= -L\left(\frac{\partial w_s}{\partial T} dT + \frac{\partial w_s}{\partial p} dp\right) &(2)\\
\left(c_p + L\frac{\partial w_s}{\partial T}\right) dT &= \left(\alpha + L\frac{\partial w_s}{\partial p}\right) dp &(3)\\
\frac{dT}{dp} &= \frac{\alpha + L\frac{w_s}{p}}{c_p + L\epsilon\frac{L w_s}{R_d T^2}} &(4)\\
\frac{dT}{dp} &= \frac{\alpha}{c_p}\left[\frac{1+\frac{L w_s}{R_d T}}{1+\epsilon\frac{L^2 w_s}{c_p R_d T^2}}\right] &(5)
\end{aligned}$$

we know that

$$\begin{aligned}
\Gamma_s(T,p) &= -\frac{dT}{dz} &(6)\\
&= -\frac{dT}{dp}\frac{dp}{dz} = \rho g \frac{dT}{dp} &(7)\\
&= \frac{g}{c_p}\left[\frac{1+\frac{L w_s}{R_d T}}{1+\epsilon\frac{L^2 w_s}{c_p R_d T^2}}\right] &(8)\\
&= \Gamma\left[\frac{1+\frac{L w_s}{R_d T}}{1+\epsilon\frac{L^2 w_s}{c_p R_d T^2}}\right] &(9)
\end{aligned}$$

This means that the pseudoadiabatic lapse rate is not constant, but a function of temperature and pressure. Notice that the denominator differs from the numerator by $\frac{\epsilon L}{c_p T}$ which is roughly

$1555/T$ which is larger than unity, so the denominator exceeds the numerator so $\Gamma_s < \Gamma$. This means that the temperature changes more slowly along a pseudoadiabat than along a dry adiabat. The reason is the release of latent heat by the condensation of vapor to liquid water.

- It can be shown that Lw_s/R_dT increases with increasing temperature (because w_s increases with T), so Γ_s decreases with increasing temperature.
- Also, because w_s decreases with increasing pressure, Γ_s increases with increasing pressure.
- The slope of a pseudoadiabat increases and approaches that of a dry adiabat as pressure and temperature decrease.

2) Derivation of Equivalent Potential Temperature

Analogous to the potential temperature, which is conserved along the dry adiabat, the quantity that is conserved along a pseudoadiabat is the equivalent potential temperature. We begin, again, with the First Law (Equation 1), use $p\alpha = RT$ and divide by T.

$$c_p \frac{dT}{T} - R_d \frac{dp}{p} = \frac{-Ldw_s}{T} \tag{10}$$

We can also take the ln of the potential temperature $\ln\theta = \ln T + \kappa \ln p_0 - \kappa \ln p$, differentiating and multyplying by c_p we have:

$$c_p \frac{d\theta}{\theta} = c_p \frac{dT}{T} - R_d \frac{dp}{p} \tag{11}$$

which means that

$$\frac{-L}{c_pT} dw_s = \frac{d\theta}{\theta} \tag{12}$$

we can demonstrate that $\frac{L}{c_pT} dw_s \approx d\left(\frac{Lw_s}{c_pT}\right)$, so we can integrate to obtain:

$$d\left(\frac{Lw_s}{c_pT}\right) + d\ln\theta = d\ln\left[\theta exp\left(\frac{Lw_s}{c_pT}\right)\right] = 0 \tag{13}$$

Integrated to obtain

$$\theta_e = \theta exp\left(\frac{Lw_s}{c_pT}\right) = constant \tag{14}$$

The equivalent potential temperature is conserved along a pseudoadiabat.

3) Adiabatic Wet Bulb Temperature T_{aw}

Point (T_{aw}, p) on the $\theta_e(T_s, p_s)$ pseudoadiabat which, is given by:

$$T_{aw}\left(\frac{1000}{p}\right)^{\kappa} exp\left(\frac{Lw_s(T_{aw}, p)}{c_pT_{aw}}\right) = \theta_e(T_s, p_s) \tag{15}$$

On a skew T, it is given by the isotherm passing through the intersection of the $\theta_e(T_s, p_s)$ pseudoadiabat with the isobar p.

4) Adiabatic Wet-Bulb Potential Temperature θ_{aw}

Point $(\theta_{aw}, 1000)$ on the $\theta_e(T_s, p_s)$ pseudoadiabat given by

$$\theta_{aw} exp\left(\frac{Lw_s(\theta_{aw}, 1000)}{c_p\theta_{aw}}\right) = \theta_e(T_s, p_s) \tag{16}$$

5) Adiabatic Equivalent Temperature T_{ae}

If we follow the pseudoadiabat $(\theta_e(T_s, p_s))$ to the top of the atmosphere, all the water is condensed out and $r = r_s = 0$. Because the air is completely dry, the pseudoadiabat is equal to the dry adiabat given by:

$$T_{ae}\left(\frac{1000}{p}\right)^{\kappa} = \theta_e(T_s, p_s) \tag{17}$$

$$T_{ae}\left(\frac{p}{1000}\right)^{\kappa} \theta_e(T_s, p_s) \tag{18}$$

On a SkewT, follow the pseudoadiabat up to the lowest pressure, then down to the dry adiabat until it intersects the p isobar.

6) Adiabatic Equivalent Potential Temperature θ_{ae}

The temperature that crosses the dry adiabat passing through T_{ae} at p=1000.

$$\theta_{ae} = \theta_e(T_s, p_s) \tag{19}$$

Obtained by following the pseudoadiabat to the lowest pressure on the chart and down the dry adiabat through this point to the 1000 mb isobar. This is the way to determine the value of $\theta_e(T_s, p_s)$ on a skew T, log p graph.