

1. Ideal Gas Law

It is convenient to express the amount of a gas as the number of moles n . One mole is the mass of a substance that contains 6.022×10^{23} molecules (N_A , Avogadro's number). $n = m/M$ where m is the mass of a substance and M is the molecular weight.

Boyle's Law: For constant temperature. $pV = \text{constant}$.

Charle's First Law: For constant pressure. $V/T = \text{constant}$.

Charle's Second Law: For constant volume. $P/T = \text{constant}$.

These all come together as the ideal gas law:

$$\begin{aligned} pV &= nR^*T \\ &= \frac{m}{M}R^*T \\ &= mRT \end{aligned} \tag{1}$$

where R^* is the universal gas constant 8.3145 J K^{-1} , p is pressure, T is temperature, M is molar weight of the gas, V is volume, m is mass and $n = m/M$ is the molar abundance of a fixed collection of matter (an air parcel). The specific gas constant R is related to the universal gas constant R^* as $R = R^*/M$. The form of the gas law that doesn't depend on the dimensions of the system is $p = \rho RT$ and $p\nu = RT$ where $\nu = 1/\rho$.

A mixture of gases obeys similar relationships as do its individual components. The partial pressure p_i of the i th component-the pressure the i th component would exert in isolation at the same volume and temperature as the mixture-satisfies the equation:

$$p_i V = m_i R_i T \tag{2}$$

where R_i is the specific gas constant of the i th component. The partial volume V_i -the volume that the i th component would occupy in isolation at the same pressure and temperature as the mixture-satisfies the equation:

$$p V_i = m_i R_i T \tag{3}$$

Dalton's law asserts that the pressure of a mixture of gases equals the sum of their partial pressures, and the volume of the mixture equals the sum of the partial volumes.

$$p = \sum_i p_i \tag{4}$$

$$V = \sum_i V_i \tag{5}$$

The equation of state for the mixture can be obtained by summing over all of the components:

$$pV = T \sum_i m_i R_i \tag{6}$$

Then, defining the mean specific gas constant

$$\bar{R} = \frac{\sum_i m_i R_i}{m} \quad (7)$$

We can obtain the equation for the mixture

$$pV = m\bar{R}T \quad (8)$$

We can also define the molar weight of the mixture as:

$$\bar{M} = \frac{\sum_i n_i M_i}{\sum_i n_i} = \frac{m}{n} \quad (9)$$

and

$$\bar{R} = \frac{R^*}{\bar{M}} \quad (10)$$

For dry air (air without water vapor) the ideal gas law becomes:

$$p_d = \rho_d R_d T \quad (11)$$

where $\alpha_d = 1/\rho_d$. and R_d is the gas constant for 1 kg of dry air. For dry air $\bar{M} = M_d = 28.97g$ (you will do this in HW 2).

$$R_d = \frac{R^*}{M_d} = \frac{8.3145}{.02897} = 287 JK^{-1}kg^{-1} \quad (12)$$

For water vapor the ideal gas law is $e\alpha_v = R_v T$. $M_w = 18.016g$ and $R_v = 461.51 JK^{-1}kg^{-1}$.

$$\frac{R_d}{R_v} = \frac{M_w}{M_d} = \epsilon = 0.622 \quad (13)$$

a. Virtual Temperature

When there is a mixture of water vapor and dry air, it is convenient to retain the gas constant for dry air and use a fictitious temperature called virtual temperature in the ideal gas equation. The density of a mixture of dry air and water vapor is:

$$\rho = \frac{m_d + m_v}{V} = \rho_d + \rho_v = \frac{p - e}{R_d T} + \frac{e}{R_v T} = \frac{p}{R_d T} \left[1 - \frac{e}{p}(1 - \epsilon) \right] \quad (14)$$

$$p = \rho R_d T_v \quad (15)$$

where

$$T_v \equiv \frac{T}{1 - \frac{e}{p}(1 - \epsilon)} \quad (16)$$

Virtual temperature is the temperature that dry air would have to attain in order to have the same density as the moist air at the same pressure. Because moist air is less dense than dry air at the same temperature and pressure, the virtual temperature is always greater than the actual

temperature by a few degrees.

2. Stratification of Mass

If vertical accelerations are ignored, Newton's second law of motion applied to a column of air between pressure p and $p + dp$ reduces to a balance between the weight of that column and the net pressure force acting on it

$$pdA - (p + dp)dA = \rho g dV \quad (17)$$

which reduces to:

$$\frac{dp}{dz} = -\rho g \quad (18)$$

Integrating between height z and the top of the atmosphere (TOA)

$$-\int_{p(z)}^{p(\infty)} dp = \int_z^{\infty} g\rho dz \quad (19)$$

$$p(z) = \int_z^{\infty} g\rho dz \quad (20)$$

This implies that the pressure at height z is equal to the weight of the air in the vertical column of unit cross sectional area lying above that level.

which is known as *hydrostatic balance*. This is a good approximation even if the atmosphere is in motion because vertical displacements of air and their time derivatives are small compared to the forces in 18.

3. Geopotential

Work that must be done against the earth's gravitational field to raise a mass of 1kg from sea level to that point (gravitational potential per unit mass). The work (in Joules) in raising 1kg from z to $z + dz$ is gdz , therefore,

$$d\Phi \equiv gdz = -\alpha dp \quad (21)$$

Integrating

$$\Phi(z) = \int_0^z gdz \quad (22)$$

The geopotential doesn't depend on the path through which the unit mass is taken in reaching that point. We define the *geopotential height* Z as

$$Z \equiv \frac{\Phi(z)}{g_0} = \frac{1}{g_0} \int_0^z gdz \quad (23)$$

g_0 is the globally averaged acceleration due to gravity (9.81 m s^{-2}). z and Z are almost the

same in the lower atmosphere where $g_0 \approx g$. It is not convenient to deal with the density of a gas....we usually use:

$$\frac{dp}{dz} = -\frac{pg}{RT} = -\frac{pg}{R_d T_v} \quad (24)$$

$$d\Phi = g dz = -R_d T_v \frac{dp}{p} \quad (25)$$

$$\int_{\Phi_1}^{\Phi_2} d\Phi = \int_{p_1}^{p_2} -R_d T_v \frac{dp}{p} \quad (26)$$

$$\Phi_1 - \Phi_2 = -R_d \int_{p_1}^{p_2} T_v \frac{dp}{p}$$

$$Z_2 - Z_1 = \frac{R_d}{g} \int_{p_1}^{p_2} T_v \frac{dp}{p}$$

This is called the *hypometric equation*. This equation can also be expressed as:

$$Z_2 - Z_1 = \bar{H} \ln \left(\frac{p_1}{p_2} \right) = \frac{R_d \bar{T}_v}{g} \ln \left(\frac{p_1}{p_2} \right) \quad (27)$$

where

$$\bar{T}_v \equiv \frac{\int_{p_2}^{p_1} T_v \frac{dp}{p}}{\ln \left(\frac{p_1}{p_2} \right)} \quad (28)$$

The difference $Z_2 - Z_1$ is referred to as the geopotential thickness of the layer between these two pressure levels.

For an *isothermal* atmosphere, neglecting the virtual temperature correction,

$$Z_2 - Z_1 = H \ln(p_1/p_2) \quad (29)$$

or

$$p_2 = p_1 \exp \left[-\frac{Z_2 - Z_1}{H} \right] \quad (30)$$

where

$$H \equiv \frac{RT_v}{g_0} \quad (31)$$

H is the *scale height* which represents the characteristic vertical dimension of the mass distribution, or the e -folding depth. H ranges from 7 to 8km in the lowest 100km of the atmosphere. Global mean pressure and density decrease with altitude approximately exponentially, from about 1000mb at the surface to only about 100mb at 15km. 90% of the atmosphere's mass lies beneath this level. Turbulent mixing below 100km makes the densities of passive constituents decrease

with altitude at the same exponential rate, which gives air a homogeneous composition with constant mixing ratios. $r_{N_2} \approx .78$, $r_{O_2} \approx .21$, $M_d = 28.96 \text{ gmol}^{-1}$, $R_d = 287.05 \text{ Jkg}^{-1} \text{ K}^{-1}$.

Above 100km, the mean free path quickly becomes larger than turbulent displacements of air. Turbulent air motions are strongly damped by diffusion of momentum and heat and diffusive transport becomes the dominant mechanism for transporting properties vertically.

Pressure decreases monotonically with height, so pressure surfaces never intersect. Because of hydrostatic equilibrium, the elevation of an isobaric surface may be interpreted analogously to the pressure on a surface of constant altitude. Therefore, centers of low elevation in the height of a 500mb surface imply centers of low pressure on a constant altitude surface.

In mountainous terrain, differences in pressure are largely due to differences in elevation. These differences must be corrected to isolate the changes in pressure due to the passage of weather systems. For the layer between the Earth's surface (g) and sea level (0) the hypsometric equation becomes

$$Z_g = \bar{H} \ln \frac{p_0}{p_g} \quad (32)$$

and

$$p_0 = p_g \exp \left(\frac{g_0 Z_g}{R_d \bar{T}_V} \right) \quad (33)$$

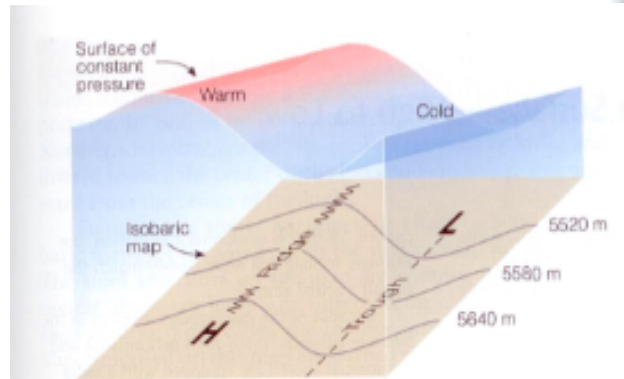
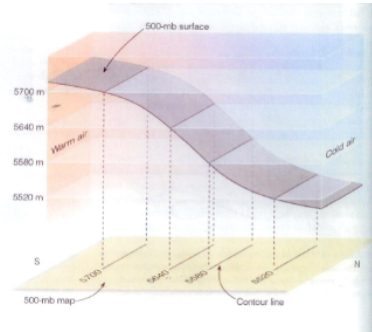
Radiosondes measure $T(p)$. Measurements are made twice per day at all of the world's meteorological stations at 0Z and 12Z. The heights calculated for a given pressure for all stations are then entered onto a map.

List of Figures

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• FIGURE 8.14 Changes in elevation of an isobaric surface (500 mb) show up as contour lines on an isobaric (500 mb) map. Where the surface dips most rapidly, the lines are closer together.



• FIGURE 8.15 The wavelike patterns of an isobaric surface reflect the changes in air temperature. An elongated region of warm air aloft shows up on an isobaric map as higher heights and a ridge; the colder air shows as lower heights and a trough.

FIG. 1. Ahrens