Contents

1. Soil Moisture

Porosity depends on particle size distribution and structure.

$$n_0 = \frac{\text{Volume of Voids}}{\Delta\forall} \tag{1}$$

Volumetric water content. When soil is fully saturated θ_s=n_0.

$$\theta = \frac{\text{Volume of water in } \Delta\forall}{\Delta\forall} \tag{2}$$

1a. *Continuity Equation*

The continuity equation for a control volume containing unsaturated soil with dimensions $\delta x, \delta y, \delta z$ with flow only in the vertical direction is:

Consequently:

$$\frac{\partial\theta}{\partial t} = -\frac{\partial q_i}{\partial x_i} \tag{3}$$

This is the continuity equation for one-dimensional unsteady unsaturated flow in a porous medium. It is applicable at shallow depths, at greater depth, changes in water density and in the porosity must be taken into account. In this equation q is the volumetric flow rate per unit area of soil, this is called the Darcy flux:

1b. *Darcy's Law*

Infiltration and redistribution are flows in saturated and unsaturated porous media (soils) that are described by Darcy's Law. Darcy stated that the rate of flow of water is proportional to the change of head, the cross-sectional area and inversely proportional to the path-length of travel (and a constant of proportionality K).

$$Q = \frac{KA(h_1 - h_2)}{\Delta L} \tag{4}$$

For an infinitesimal volume, we can write the equation:

$$q_i = -K\frac{\partial h}{\partial x_i} \tag{5}$$

where

$$h = z + \frac{p}{\rho_w g} = z + \frac{p}{\gamma_w} = z + \psi_w \tag{6}$$

where ψ is the equivalent height of water.

$$\frac{\partial h}{\partial x_i} = \delta_{j3}\frac{\partial x_j}{\partial x_i} + \frac{\partial \psi_w}{\partial x_i} \tag{7}$$

so q_i

$$q_i = -K\left[\delta_{j3}\frac{\partial x_j}{\partial x_i} + \frac{\partial \psi_w}{\partial x_i}\right] \tag{8}$$

Where q_i is the volumetric flow rate in the i direction per unit cross-sectional area of the medium (m/s), z is the elevation above an arbitrary datum, p is the water pressure, γ_w is the weight density of water and K is the hydraulic conductivity of the medium. Darcy's Law describes the flow at a *representative elemental volume* of the soil that includes pore spaces and soil particles. Flow occurs because of gradients in gravitational potential energy (term one) and pressure potential energy (term two).

If we consider only flows in the vertical (z) direction:

$$q_z = -K\left[\frac{\partial z}{\partial z} + \frac{\partial (\psi_w}{\partial z}\right] \tag{9}$$

$$= -K\left[1 + \frac{\partial \psi_w}{\partial z}\right] \tag{10}$$

The magnitude of the gravitational potential energy gradient will always equal one (+ or - depending on the direction of flow and definition of the coordinate system, in this case +1 going up and -1 going down). p ≤ 0 for the unsaturated flows considered here.

1c. *True Velocity*

While q_i is the volumetric flow rate per unit bulk area, the true velocity

$$v_{qi} = \frac{q_i}{n_0} \tag{11}$$

for fully saturated conditions and

$$v_{qi} = \frac{q_i}{\theta} \tag{12}$$

for unsaturated conditions.

We have until now assumed that K is independent of direction (isotropic). In reality it isn't and K is actually a tensor. For the most general case:

$$q_i = -K_{ij}\frac{\partial h}{\partial x_j} \tag{13}$$

In the derivations that follow we will assume isotropic conditions.

It is convenient to use a pressure head ψ (units of m) defined as

i. Pressure It is conventional to measure pressure relative to atmospheric pressure. $p > 0$ and $\psi > 0$ in saturated flows and $p < 0$ and $\psi < 0$ in unsaturated flows, The *water table* is the surface at which $p = 0$. Negative pressure is called *tension or suction* and ψ is called the *tension head, matric potential or matric suction* when $p < 0$. In unsaturated soils, water is held to the mineral grains by surface tension forces. When talking about infiltration, p and ψ will always be negative.

The relation between the pressure head ψ and water content θ is called the *moisture-characteristic curve*. The relationship is highly nonlinear. Pressure head is zero when water content equals porosity. There is a point when significant volumes of air begin to appear in the soi and this is the air-entry tension ψ_{ae}. Beyond this point, water content begins to decrease rapidly and more gradually. After a certain value, even very large tensions will not dry out the soil because

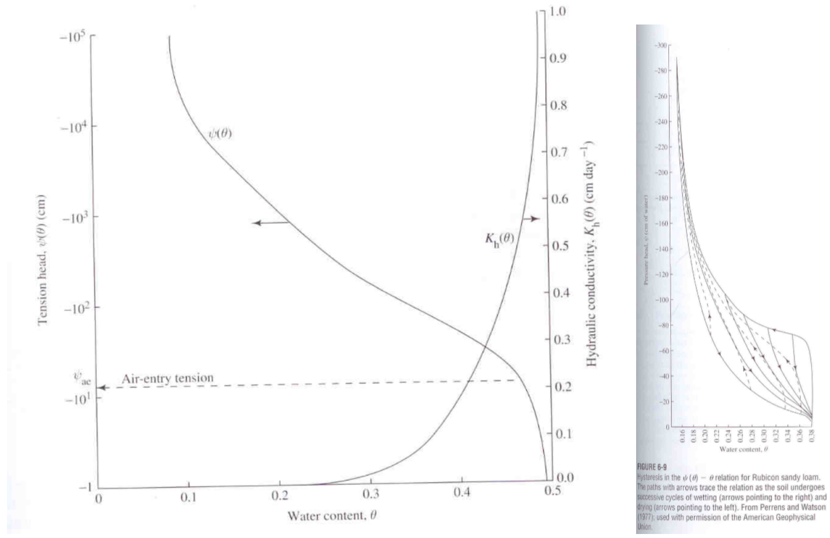

Figure 1: Dingman Figure 6-7 and 6-9

this water content is very tightly held in the soil pores by capillary and electrochemical forces. Given a certain degree of saturation, tension is much higher in finer-grained soils than in coarse grained soils.

In reality the value of tension at a given water content is not unique, but depends on the soil's history of wetting and drying - however, this *hysteresis* is difficult to model and not usually incorporated in hydrologic models.

ii. Hydraulic Conductivity Hydraulic conductivity is the rate at which water moves through a porous medium under a unit potential energy gradient. Under saturated conditions, this size is determined by the soil-grain size. For unsaturated conditions it is determined by grain size and degree of saturation. K is very low at low to moderate water content, and increases nonlinearly to its saturated value (K^*) as water content increases. K increases by several orders of magnitude in going from clay to silty clay loam to sand (also depending on degree of saturation).

iii. Incorporating K - θ and ψ - θ Relations into Models Brooks and Corey (1964), Campbell (1974) and Van Genuchten (1980) have proposed various relations for K - θ and ψ - θ relations:

Chapter 6 • Water in Soils: Infiltration and Redistribution

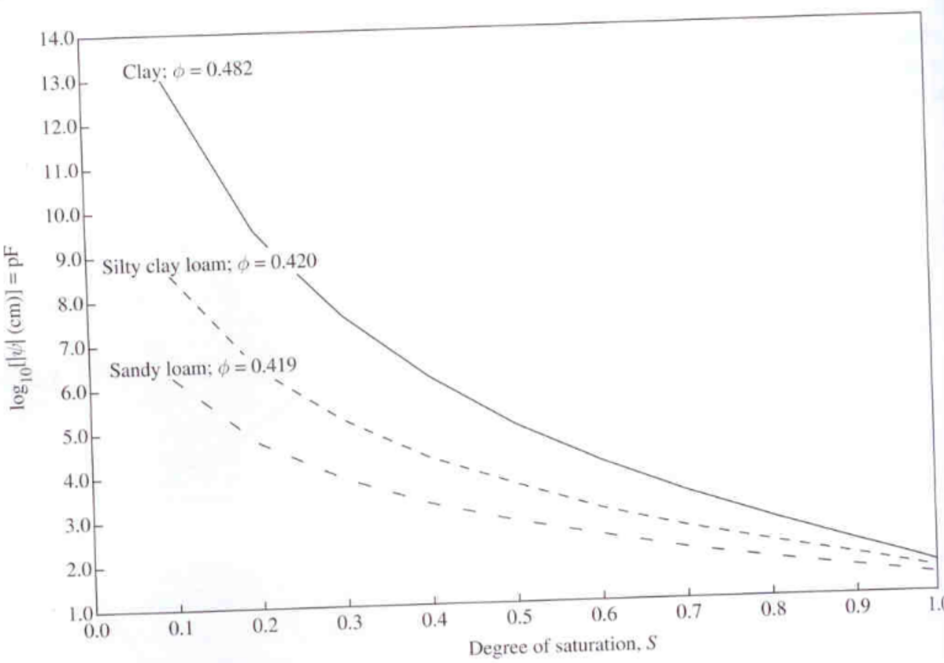

FIGURE 6-8
Soil-water pressure (tension), ψ, vs. degree of saturation, S, for soils of three different textures. Note that the vertical axis gives the base-10 logarithm of the absolute value of the pressure (which is negative), expressed in cm of water (pF). Curves are based on typical values given by Clapp and Hornberger (1978). The sandy-loam curve is for the soil discussed in Examples 6-1–6-3.

Figure 2: Dingman Figure 6-8

$$|\psi(\theta)| = |\psi_{ae}| \left(\frac{n_0}{\theta}\right)^b \tag{14}$$

$$K(\theta) = K^* \left(\frac{\theta}{n_0}\right)^c \tag{15}$$

These equations ignore hysteresis, apply only to $|\psi| \geq |\psi_{ae}|$ - crude approximations for $|\psi| \leq |\psi_{ae}|$ can be made by a straight line from $|\psi| = 0$ to $|\psi| = |\psi_{ae}|$. b is the pore size distribution index, c is the pore-disconnectedness index.

$$c = 2b + 3 \tag{16}$$

Typical values determined by statistical analysis of data for a large number of soils are given below:

6.4 Water Conditions in Natural Soils **235**

TABLE 6-1
Representative values of parameters in Equations (6-12) and (6-13) based on analysis of 1845 soils; values in parentheses are standard deviations. Data from Clapp and Hornberger (1978).

Soil Texture	ϕ	K_h^* (cm s^{-1})	$\lvert\psi_{ae}\rvert$ (cm)	b	
Sand	0.395 (0.056)	1.76×10^{-2}	12.1 (14.3)	4.05 (1.78)	
Loamy sand	0.410 (0.068)	1.56×10^{-2}	9.0 (12.4)	4.38 (1.47)	
Sandy loam	0.435 (0.086)	3.47×10^{-3}	21.8 (31.0)	4.90 (1.75)	.419
Silt loam	0.485 (0.059)	7.20×10^{-4}	78.6 (51.2)	5.30 (1.96)	
Loam	0.451 (0.078)	6.95×10^{-4}	47.8 (51.2)	5.39 (1.87)	
Sandy clay loam	0.420 (0.059)	6.30×10^{-4}	29.9 (37.8)	7.12 (2.43)	
Silty clay loam	0.477 (0.057)	1.70×10^{-4}	35.6 (37.8)	7.75 (2.77)	.420
Clay loam	0.476 (0.053)	2.45×10^{-4}	63.0 (51.0)	8.52 (3.44)	
Sandy clay	0.426 (0.057)	2.17×10^{-4}	15.3 (17.3)	10.4 (1.64)	
Silty clay	0.492 (0.064)	1.03×10^{-4}	49.0 (62.1)	10.4 (4.45)	
Clay	0.482 (0.050)	1.28×10^{-4}	40.5 (39.7)	11.4 (3.70)	[illegible]

Figure 3: Table 6-1 Dingman

iv. Hydraulic Diffusivity It is sometimes useful to use hydraulic diffusivity $D(\theta)$ as

$$D(\theta) \equiv K(\theta) \frac{\partial \psi(\theta)}{\partial \theta} \tag{17}$$

notice the dimensions: $[m^2/s]$. This means that the flow due to the pressure gradient can be expressed as the product of the hydraulic diffusivity and the water-content gradient. Using the relationships above:

$$D(\theta) = -b\psi_{ae}K^* n_0^{-b-3}\theta^{b+2} \tag{18}$$

1d. *Richard's Equation*

We can write the continuity equation using Darcy Flux in terms of the hydraulic conductivity and hydraulic diffusivity:

$$\begin{aligned} q_i &= -K(\theta)\left[\delta_{j3}\frac{\partial x_j}{\partial x_i} + \frac{\partial \psi_w}{\partial \theta}\frac{\partial \theta}{\partial x_i}\right] && (19) \\ &= -K(\theta)\delta_{j3}\frac{\partial x_j}{\partial x_i} - D(\theta)\frac{\partial \theta}{\partial x_i} && (20) \\ & && (21) \end{aligned}$$

Darcy's Law for the vertical direction:

$$\begin{aligned} q &= -K(\theta)\left[1 + \frac{\partial \psi(\theta)}{\partial z}\right] && (22) \\ &= -K(\theta) - D(\theta)\frac{d\theta}{dz} && (23) \end{aligned}$$

This equation is exactly the same (physically) as the original Darcy, but it simplifies the following analytical solutions. Remember the original conservation equation:

$$\begin{aligned} \frac{\partial \theta}{\partial t} &= -\frac{\partial q_i}{\partial x_i} && (24) \\ & && (25) \end{aligned}$$

We can now express this in terms of the Darcy Flux, to obtain the **Three Dimensional Richards Equation for Isotropic Conditions**.

$$\frac{\partial \theta}{\partial t} = -\frac{\partial}{\partial x_i}\left[-K(\theta)\delta_{j3}\frac{\partial x_j}{\partial x_i} - D(\theta)\frac{\partial \theta}{\partial x_i}\right] \tag{26}$$