The Temperature of Planets: Radiative Equilibrium Temperature

Any object with a finite temperature emits radiation according to Planck's law. The radiative *emission* from a blackbody per unit surface area with a temperature, T, in Kelvin is given by the Stephan Boltzmann law (which is the spectral integral of Planck function):

$$F = \sigma T^4 \quad (\mathrm{W/m^2}) \tag{1}$$

where σ is the Stephan-Boltzmann constant: 5.67×10^{-8} W/m²/K⁴. So the total radiative emission from a blackbody depends only on its temperature, a rather remarkably simple relation.

The Sun has a blackbody temperature of 5,780 K. Therefore the flux at the Sun's surface is 6.3×10^{23} W/m².

This flux is the energy passing through each square meter of the Sun's outer "surface" each second. To obtain the total emission from an object we multiply this watts per unit area by the object's surface area which for a sphere is $4\pi r^2$ where r is the radius of the sphere.

$$F_{tot} = \sigma T^4 \, 4\pi r^2 \qquad (\mathrm{W}) \tag{2}$$

The Sun's radius is 6.96×10^5 km (quick memory point: Earth's average radius is 6,378 km, Jupiter's radius is about 10 times that, and the Sun's is about 10 times that). So the total power radiated by the sun is 3.9×10^{42} W.

The Sun's radiation spreads out over space until it reaches a planet such as Earth. This emission spreads over a surface area of the sphere whose radius is the distance from the sun to the planet. The solar flux at the planet is the total flux radiated by the Sun divided by a sphere of radius the planet's orbital radius.

$$F_{planet} = \frac{F_{sun-total}}{4\pi R^2_{planet-orbit}} = \frac{\sigma T^4_{sun} 4\pi r^2_{sun}}{4\pi r^2_{planet-orbit}} = \sigma T^4_{sun} \frac{r^2_{sun}}{r^2_{planet-orbit}} \qquad (\mathrm{W/m^2}) \tag{3}$$

The amount of radiation striking the planet is this flux times the crossectional area of the planet:

$$F_{tot-planet} = \sigma T^4_{sun} \frac{r^2_{sun}}{r^2_{planet-orbit}} \pi r^2_{planet} \qquad (\mathrm{W}) \tag{4}$$

Only a portion of this radiation striking the planet is absorbed. The fraction that is reflected is called the albedo, A. So the amount of solar radiation power that is absorbed by the planet is

$$F_{tot-planet-absorbed} = F_{tot-planet}\left(1 - A_{planet}\right) = \sigma T_{sun}^4 \frac{r_{sun}^2}{r_{planet-orbit}^2} \pi\, r_{planet}^2 \left(1 - A_{planet}\right) \qquad \text{(W)} \tag{5}$$

So (5) defines the solar energy absorbed by the planet. In radiative equilibrium, this absorbed energy is balance by the energy emitted by the planet. This balance is written as

$$\sigma T_{sun}^4 \frac{r_{sun}^2}{r_{planet-orbit}^2} \pi\, r_{planet}^2 \left(1 - A_{planet}\right) = \sigma T_{planet}^4\, 4\pi\, r_{planet}^2 \qquad \text{(W)} \tag{6}$$

So the *radiative equilibrium* temperature of the planet, T_{planet}, is given as

$$T_{planet} = T_{sun}\left[\frac{r_{sun}}{2 r_{planet-orbit}}\right]^{1/2} \left(1 - A_{planet}\right)^{1/4} \qquad \text{(K)} \tag{7}$$

Notice that it does not depend on the size of the planet. Rather it depends on the size and temperature of the sun and the planet's distance from the sun and albedo.

Note that the temperatures of the gas giant planets tend to be somewhat higher than this equilibrium temperature because as the planet contracts over time, the energy gained by compression is radiated out to space.

If the planet has no atmosphere, this radiative equilibrium temperature equals the surface temperature of the planet. If the planet has a substantial atmosphere (surface pressure > 100 mb), then this is the temperature near the tropopause of the planet approximately the level in the atmosphere where the radiation from the planet is emitted to space.

Tsun	5780	K
Rsun	6.960E+05	km
Solar flux density	6.328E+23	W/m^2
total solar flux	3.852E+42	W

	Mercury	**Venus**	**Earth**	**Mars**	**Jupiter**	**Titan**	**Triton**	**Units**
Albedo	0.1	0.7	0.3	0.15	0.45	0.22	0.7	
Rsun-planet	5.50E+07	1.10E+08	1.50E+08	2.28E+08	7.70E+08	1.43E+09	4.49E+09	**km**
Rsp(AU)	0.368	0.736	1.000	1.524	5.151	9.580	30	**AU**
Solar flux	10134.15	2533.54	1371.61	590.82	51.70	14.95	1.52	**W/m^2**
Solar flux absorbed	9120.74	760.06	960.13	502.20	28.44	11.66	0.46	**W/m^2**
Solar flux abs in troposphere						2.33		
Tequilibrium	448	241	255	217	106	85	38	**K**
Tsurf	448	740	288	227 (186-268)	NA	93	38	**K**
Tdiff	0	499	33				0	**K**
Greenhouse	no	YES	yes	some			no	
observed Te					124.4			**K**
Finterior/solar					0.909987			

The (different forms of the) Planck function

The Planck function describes the electromagnetic energy spectrum emitted by a perfect blackbody, that is a perfect absorber and emitter in equilibrium with its radiation field as would be the case in an oven. The Planck function can be written in terms of frequency, ν, or wavelength, λ. There are other versions as well and you must check the units to understand the form. The radiance form as a function of frequency with units of Watts/steradian/m^2/Hz is

$$B(\nu,T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

Check units: J s $s^2/(s^3 m^2)$ = J/m^2 = J/m^2/s/Hz = W/m^2/Hz. The problem is you can't see the steradians