1. Convert between moisture variables.
Conditions: Temperature = 75°F. Dew point = 60°F, Pressure = 1000 mb.

Convert temperatures to Kelvin: $T(K) = \dfrac{5(T(F)-32)}{9} + 273.15$

T = 297.14 K, T_d = 288.81 K

Find the following

a. water vapor partial pressure

$\boldsymbol{e = e_s(T_d)}$. Using the Groff Gratch equation below, or a simpler form from the http://cires.colorado.edu/~voemel/vp.html like the Bolton (1980) approximation:

$$e_s = 6.112 \exp[17.67 * T / (T+243.5)]$$

$$e = 6.112 \exp[17.67 * T_d / (T_d + 243.5)]$$

where T is in [°C] and e_s in [hPa]

The answer is **e = 17.8 mb**

b. water vapor saturation vapor pressure

$e_s = e_s(T_d)$. Using the Groff Gratch equation below or the Bolton equation yields $\boldsymbol{e_s}$ **= 29.8 mb**

c. relative humidity

RH = $\boldsymbol{e/e_s}$ **= 17.8/29.8 = 59.6%**

d. specific humidity

specific humidity is the mass of water vapor per mass of moist air: $q = \dfrac{em_v}{(P-e)m_d + em_v}$.

Plugging in values we get **0.0111 g/g or 11.1 g/kg**

e. saturation specific humidity

specific humidity is the mass of water vapor that would be in the air at saturation per mass of the actual (moist) air: $q = \dfrac{e_s m_v}{(P-e)m_d + em_v}$.

Plugging in values we get **0.0186 g/g or 18.6 g/kg**

f. absolute humidity

mass density of water vapor in the air. $e = \rho_v R_v T$ so $\rho_v = e / R_v T$

The answer is **12.9 grams per cubic meter**.

2. *The vertical gradient of dew point temperature in a well mixed boundary layer*
   a. *Show that* $\frac{dT_d}{dz} = -\frac{g}{L}\frac{m_d T_d^2}{m_v T} \sim -\frac{g}{L}\frac{m_d T_d}{m_v}$

Derivation:

First, for a well mixed boundary layer

$$\frac{e}{P} = \text{const}$$

$$\frac{de}{e\,dz} = \frac{dP}{Pdz} = -\frac{1}{H} = \frac{mg}{R^*T} \cong \frac{g}{RT}$$

Second, to relate e and T_d we need the Clausius Clapeyron equation since $e = e_s(T_d)$:

$$\frac{de_s}{e_s} = \frac{L\,dT}{R_v T^2}$$

$$\frac{de}{e} = \frac{L\,dT_d}{R_v T_d^2}$$

$$\frac{de}{e\,dz} = -\frac{g}{RT} = \frac{L}{R_v T_d^2}\frac{dT_d}{dz}$$

$$\frac{dT_d}{dz} = -\frac{g}{RT}\frac{R_v T_d^2}{L} = -\frac{g}{L}\frac{m_d T_d^2}{m_v T} \sim -\frac{g}{L}\frac{m_d T_d}{m_v}$$

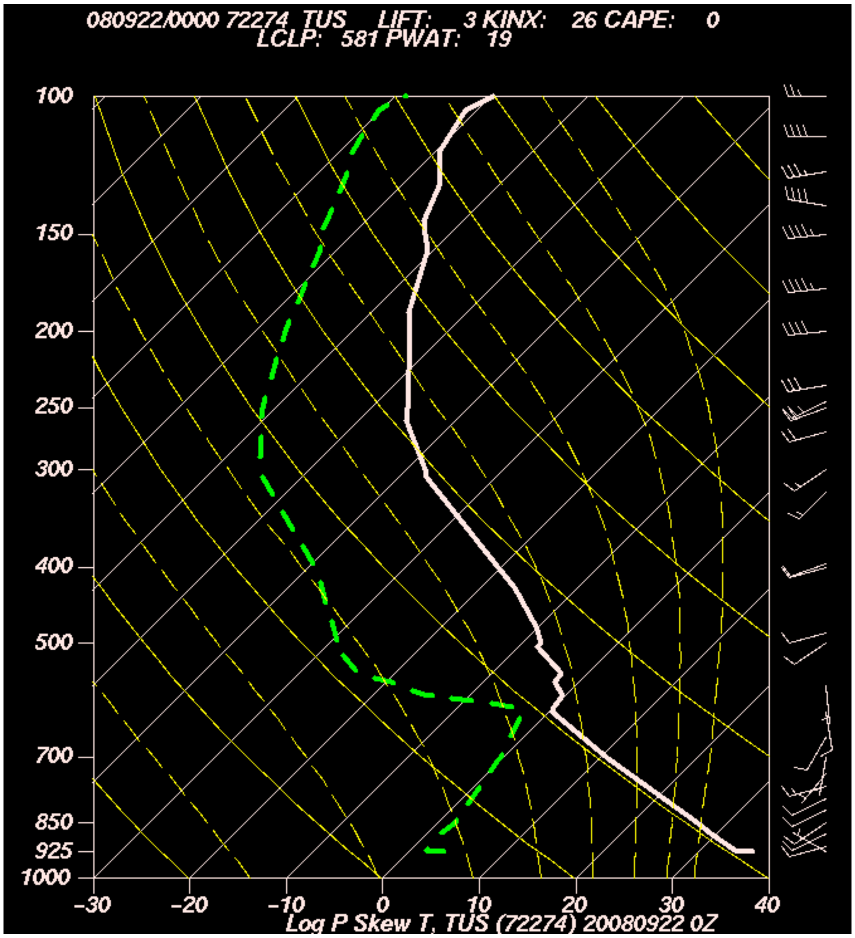
where the last step is because T_d is close to T in units of K.

   b. *Show that the value of* dT_d/dz *is typically in the range from -1.5 to -1.7K/km.*

Random examples of dT_d/dz:

T	T_d	dT_d/dz (K/km)
288	278	-1.69
288	268	-1.57
288	258	-1.46
313	283	-1.62

3. Using the figure below, and assuming a well mixed boundary layer, if the dew point at 1000 mb had been 15°C, approximately at what altitude and pressure level would the air have been saturated such that a cloud formed there



At 3 km altitude which is about 700 mb, the dew point temperature would have been 15°C minus 3 km x 1.7 K/km ~ 10°C which is about the air temperature at 700 mb. Therefore the cloud would have formed at about 700 mb.

4. Calculate a moist adiabatic atmospheric structure.

You are going to construct approximately the moist adiabatic curve in the figure above that starts at $T=22^oC$ and $P = 1000$ mb.

a. *Create a spreadsheet with at least 5 columns titled: altitude (km), temperature(K), pressure (mb), es (mb) and dT/dz (K/km)*
b. *Setup the altitude column so that the values run from 0 km at the bottom of the sheet to 16.5 km at the top in increments of 0.5 km*
c. *Set the surface temperature = 293.15K (= 22^oC).*
d. *Set the surface pressure = 1000 mb*
e. *In the 4th column use the Groff-Gratch equation to calculate e_s:*

$$\begin{aligned}\mathrm{Log}_{10}(e_s) = & -7.90298\,(373.16/T-1) \\ & + 5.02808\,\mathrm{Log}_{10}(373.16/T) \\ & - 1.3816\times10^{-7}\,(10^{11.344\,(1-T/373.16)}-1) \\ & + 8.1328\times10^{-3}\,(10^{-3.49149\,(373.16/T-1)}-1) \\ & + \mathrm{Log}_{10}(1013.246)\end{aligned}$$

with T in [K] and e_s in [hPa=mb]

(Note: this is a complicated equation prone to error when entering it. To verify you have entered this equation properly, make sure it yields a value of ~6.1 mb for a temperature of 0^oC = 273.15 K)

The equation should produce e_s at the surface of 26.4 mb for the surface temperature of 295.15 K

f. *In the 5th column calculate the moist adiabatic lapse rate using*

$$\frac{dT_s}{dz} = -\frac{g}{C_p}\frac{\left[1+\frac{e_s}{P}\frac{L_v}{R_vT}\right]}{\left[1+\frac{L_v^2}{C_pR_vT^2}\frac{\mu_v}{\mu_d}\frac{e_s}{P}\right]} \qquad (15)$$

To make things simple, assume L_v = 2.5e6 J/kg.

Now construct the vertical atmospheric structure using dT/dz

In the row above the surface row

g. *Set the temperature in column 2 equal to the surface temperature plus the moist adiabatic dT/dz from the surface row times the height difference between the second and first row*
h. *To determine the pressure in this row use the hypsometric equation*

$$P(z_2) = P(z_1)\exp\left[-\frac{(z_2-z_1)}{H}\right] = P(z_1)\exp\left[-\frac{(z_2-z_1)g}{RT}\right]$$

using T from the surface row or the average of the T in the surface row and T in this row from item g.

i. *Using T in this row from item g, calculate e_s.*
j. *Calculate dT/dz in the 5th column*

Repeat steps g-j for the next row up. Continue until you fill all rows

Compare your results with the curve in the figure to verify they are approximately right. Your result should look similar to the following:

z	*T*		*P*	*es*	*dT/dz*
16.5	*176.63*	*-96.52*	*98.00*	*5.09622E-05*	*-9.80*
16	*181.53*	*-91.62*	*107.66*	*0.000136736*	*-9.80*
15.5	*186.43*	*-86.72*	*117.99*	*0.000339882*	*-9.79*
15	*191.32*	*-81.83*	*129.00*	*0.000791464*	*-9.79*
14.5	*196.21*	*-76.94*	*140.73*	*0.00174135*	*-9.78*
14	*201.09*	*-72.06*	*153.20*	*0.003643459*	*-9.76*
13.5	*205.95*	*-67.20*	*166.44*	*0.007285328*	*-9.73*
13	*210.79*	*-62.36*	*180.48*	*0.013972504*	*-9.68*
12.5	*215.59*	*-57.56*	*195.36*	*0.025770291*	*-9.60*
12	*220.34*	*-52.81*	*211.10*	*0.04578768*	*-9.50*
11.5	*225.01*	*-48.14*	*227.74*	*0.078460592*	*-9.35*
11	*229.59*	*-43.56*	*245.32*	*0.12976228*	*-9.15*
10.5	*234.05*	*-39.10*	*263.88*	*0.207257414*	*-8.91*
10	*238.35*	*-34.80*	*283.48*	*0.319944686*	*-8.62*
9.5	*242.50*	*-30.65*	*304.15*	*0.477902096*	*-8.29*
9	*246.47*	*-26.68*	*325.97*	*0.691826377*	*-7.93*
8.5	*250.25*	*-22.90*	*348.99*	*0.972594859*	*-7.57*
8	*253.85*	*-19.30*	*373.27*	*1.330953463*	*-7.21*
7.5	*257.28*	*-15.87*	*398.88*	*1.777371601*	*-6.85*
7	*260.54*	*-12.61*	*425.89*	*2.322045398*	*-6.52*
6.5	*263.65*	*-9.50*	*454.38*	*2.975000058*	*-6.21*
6	*266.61*	*-6.54*	*484.43*	*3.746240245*	*-5.93*
5.5	*269.45*	*-3.70*	*516.12*	*4.645910706*	*-5.68*
5	*272.18*	*-0.97*	*549.53*	*5.684445514*	*-5.45*
4.5	*274.79*	*1.64*	*584.76*	*6.872697018*	*-5.24*
4	*277.32*	*4.17*	*621.89*	*8.222043304*	*-5.05*
3.5	*279.76*	*6.61*	*661.02*	*9.744476725*	*-4.89*
3	*282.13*	*8.98*	*702.26*	*11.4526774*	*-4.74*
2.5	*284.43*	*11.28*	*745.70*	*13.36007551*	*-4.60*
2	*286.68*	*13.53*	*791.46*	*15.48090574*	*-4.48*
1.5	*288.86*	*15.71*	*839.65*	*17.83025651*	*-4.37*
1	*291.00*	*17.85*	*890.39*	*20.42411581*	*-4.28*
0.5	*293.10*	*19.95*	*943.80*	*23.27941541*	*-4.19*
0	*295.15*	*22.00*	*1000.00*	*26.41407416*	*-4.11*

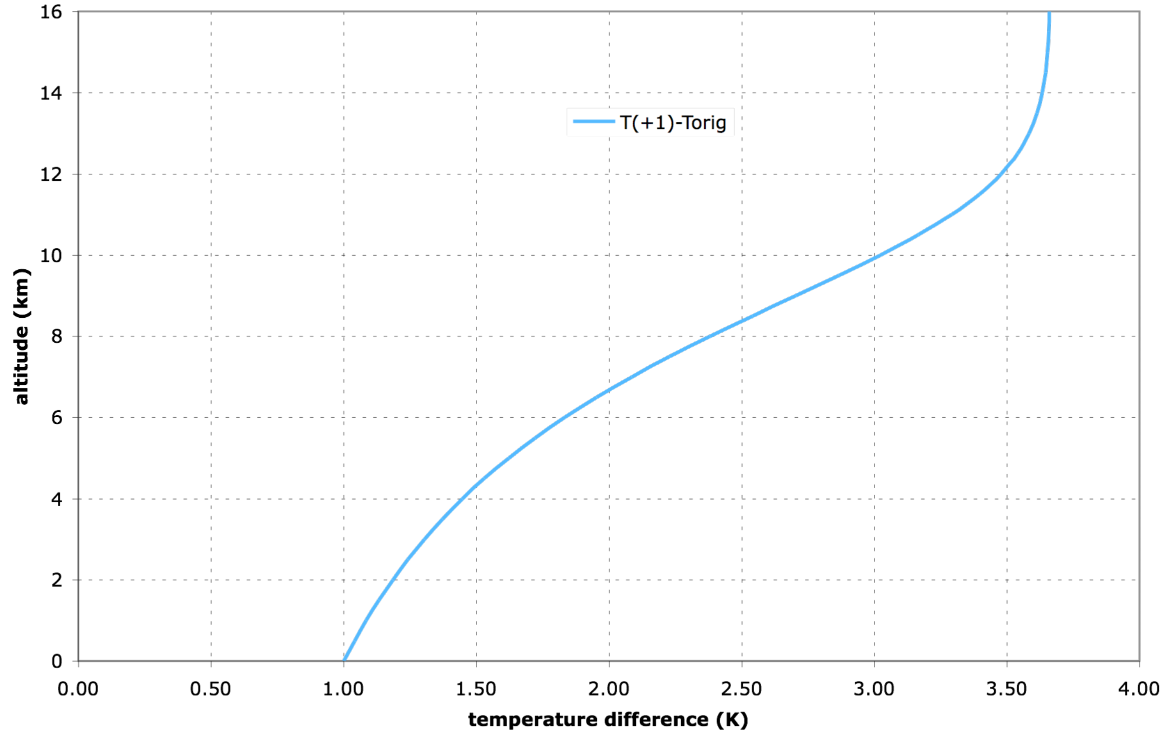
5. *Sensitivity of the moist adiabat to the surface temperature*

a. In your table, increase the surface temperature by 1K

z	T	P	es	dT/dz
16.5	180.2906455	100.57554	0.000107368	-9.80
16	185.1880509	110.2888444	0.000271763	-9.79
15.5	190.082749	120.6534585	0.000642767	-9.79
15	194.9725011	131.6951165	0.001433554	-9.78
14.5	199.8536442	143.4401531	0.003035879	-9.76
14	204.7204572	155.9156847	0.006136888	-9.73
13.5	209.5644199	169.1498783	0.011887812	-9.69
13	214.3734697	183.1723214	0.022129104	-9.62
12.5	219.131469	198.0145016	0.039660922	-9.52
12	223.8181908	213.7103715	0.068522004	-9.37
11.5	228.4101425	230.2969493	0.114210263	-9.18
11	232.8823866	247.8148705	0.1837628	-8.94
10.5	237.2111455	266.3088032	0.285633933	-8.66
10	241.3765765	285.8276647	0.429373502	-8.33
9.5	245.364928	306.4246318	0.625185854	-7.98
9	249.1695147	328.1569924	0.883494229	-7.61
8.5	252.7904306	351.0859139	1.214618593	-7.24
8	256.2333466	375.2762046	1.628615374	-6.89
7.5	259.5079031	400.7961172	2.135266567	-6.55
7	262.6261293	427.7172199	2.744171634	-6.24
6.5	265.6011384	456.1143369	3.464891123	-5.95
6	268.4461776	486.0655512	4.307103151	-5.69
5.5	271.1740091	517.6522564	5.280750069	-5.46
5	273.7965536	550.9592443	6.396165604	-5.25
4.5	276.3247188	586.0748202	7.664180811	-5.06
4	278.7683513	623.0909354	9.096211243	-4.89
3.5	281.1362619	662.1033343	10.70432908	-4.74
3	283.4362922	703.2117099	12.50132406	-4.60
2.5	285.6753999	746.519867	14.50075659	-4.48
2	287.8597502	792.1358905	16.71700552	-4.37
1.5	289.9948059	840.1723194	19.16531269	-4.27
1	292.085411	890.7463245	21.86182566	-4.18
0.5	294.1358669	943.9798899	24.82363965	-4.10
0	296.15	1000	28.0688394	-4.03

b. *Show that the temperature at 12 km has increased by about 3.5 K.*

The plot shows the difference between the two moist adiabatic structures vs. altitude where the second is 1K warmer at the surface. The temperature difference at 12 km is about 3.5 K.



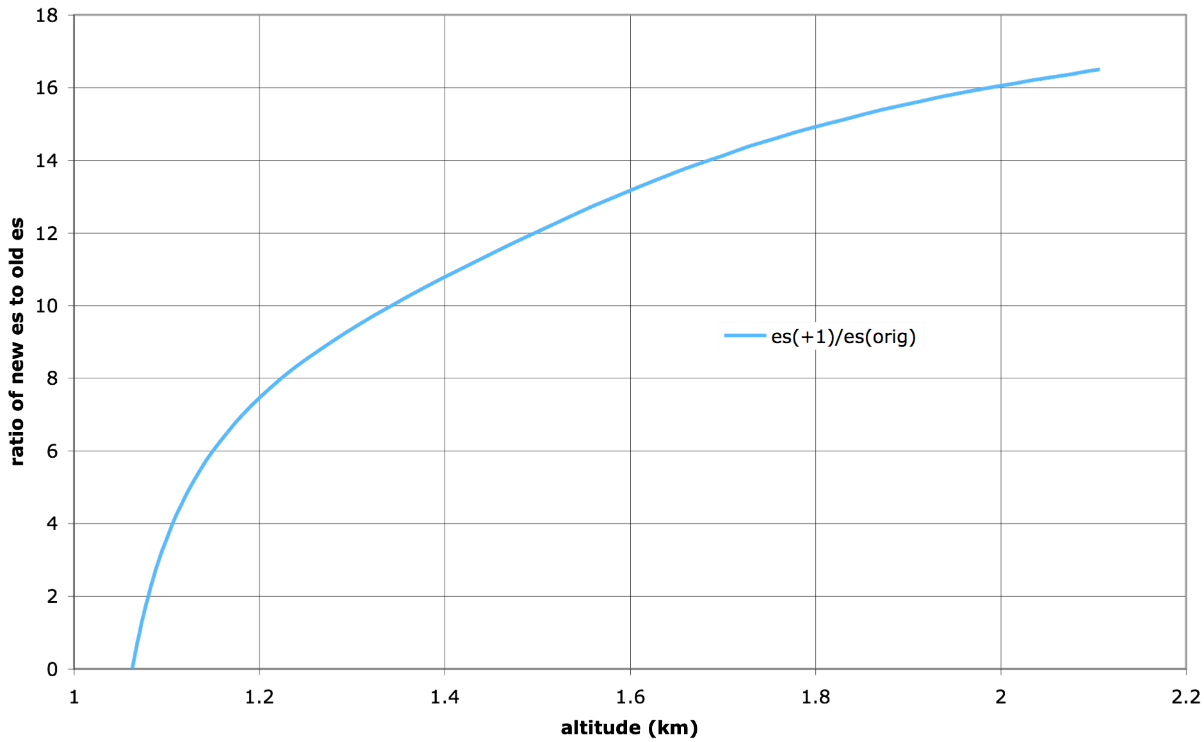
c. *Why has the upper troposphere temperature increased more than the surface temperature change*

The basic reason is the warmer the air is, the more water vapor it holds when saturated because of the Clausius Clapeyron equation. This means the air holds more latent heat that is then converted to increasing the air temperature when the air rises and the cools causing the water to be condensed. For the moist adiabat, temperatures in the upper troposphere reflect the integrated effect of having condensed almost all of the water vapor in the column. Therefore a saturated air parcel that is slightly warmer near the surface will become even warmer relative to a slightly cooler saturated parcel as it is lifted because of the additional water molecules in the slightly warmer parcel.

Specifically in this case, after lifting to 16 km, without the latent heat release, the air parcel's temperature would have been 295.15 K – 9.81 K/km*16 km = 138.2 K. The actual temperature after the lifting including the latent heat release was 181.5 K. So the latent heat release warmed the air by 43.3 K. Increasing the surface temperature by 1 K and holding the relative humidity constant should have increased the water vapor in the air by ~6.3% according to the Clausius Clapeyron equation. Therefore the latent heating should have increased by about 6.3% of 43.3K = 2.7 K. The actual increase due to latent heating was 185.2 K – 139.2 K = 46 K which is 2.7 K higher than 43.3 K.

So to the extent that the upper troposphere temperatures are controlled by the moist adiabat and that surface relative humidities don't change so the cloud base altitude doesn't change, the upper troposphere will become even warmer than the surface in a warming climate due to increased latent heat release during ascent of deep convection.

d. What is the ratio of the new e_s at 12 km to the old e_s at 12 km.



From the figure, e_s(new)/e_s(original) at 12 km is 1.5. Again, if the upper troposphere particularly in the tropics is controlled by the change in the moist adiabat, then the upper troposphere absolute humidity and vapor pressure should be increasing with time.

This moist adiabatic behavior is why climate models predict the upper troposphere should be warming faster than the lower troposphere and should be getting wetter in a fractional sense than the lower troposphere, effects that researchers are searching for in observations.