1. Pluto's orbit is far more eccentric than those of the major planets' orbits:

Aphelion: 7,375,927,931 km

Perihelion: 4,436,824,613 km

a. Determine the solar flux (watts/m^2) at each of these distances.

$$F_{planet} = \frac{F_{sun-total}}{4\pi R^2_{planet-orbit}} = \frac{\sigma T^4_{sun} 4\pi r^2_{sun}}{4\pi r^2_{planet-orbit}} = \sigma T^4_{sun} \frac{r^2_{sun}}{r^2_{planet-orbit}} \qquad (\mathrm{W/m^2})$$

Aphelion: $0.56\ W/m^2$

Perihelion: $1.56\ W/m^2$.

These are roughly 1/1,000 of the solar flux at the Earth

b. Assume the albedo is 0.7 in both cases. Determine the radiative equilibrium temperature at both distances.

So the radiative equilibrium temperature of the planet, T_{planet}*, is given as*

$$T_{planet} = T_{sun}\left[\frac{r_{sun}}{2r_{planet-orbit}}\right]^{1/2}\left(1 - A_{planet}\right)^{1/4} \qquad (K)$$

Aphelion: $29.4K$

Perihelion: $37.9K$

While these may seem like very similar small temperatures, the higher temperature is about 30% higher than the low temperature. This turns out to be very important for the size of Pluto's atmosphere because its pressure likely depends very strongly on temperature via the Clausius Clapeyron equation that we will study later that determines how much water vapor Earth's atmosphere can hold.

2. In the first figure in the 9/5/08 notes, the IR radiative flux from Earth's atmosphere into the surface is 324 watts/m^2.

a. Based on the Stephan-Boltzmann law, what is the temperature of the atmospheric level that is radiating into the surface?

$F = \sigma T^4$. => $T = (F/\sigma)^{1/4}$

$T = 274.9K = 1.8\ ^oC$

b. Assuming the Earth's surface temperature is 288 K and the atmospheric temperature decreases vertically at a rate of 6.5 K/km, at what atmospheric altitude is the IR radiation into the surface coming from ?

$z = (T-T_{surf})/(dT/dz) = (274.9-288)/(-6.5K/km) = 2\ km$

3. The outgoing IR radiation to space of 235 watts/m^2 is composed of 3 terms: 165 watts/m^2 from the atmospheric gas, 30 watts/m^2 from the atmospheric clouds and 40 watts/m^2 from the surface.

a. Take the atmospheric portion: 165 + 30 = 195 watts/m^2. Based on the Stephan-Boltzmann law, what is the temperature that is radiated from?

242.2K

b. Assuming the same atmospheric temperature structure as in the previous problem, what altitude in the atmosphere is this being radiated from?

7.05 km

c. Assume that increasing CO_2 in the atmosphere causes the atmospheric portion of the IR watts/m^2 to decrease by 4 watts/m^2, how much cooler is the new radiating temperature?

240.9 K

d. How much higher is the new radiating altitude than the original?

7.24 km => 190 m higher in altitude

4. Assuming the vertical temperature gradient remains at 6.5 K/km, how much must the surface temperature increase to bring the Earth back into equilibrium?

To get the outgoing IR back in balance with the incoming solar, the original temperature profile must shift upwards by 0.19 km. Therefore the original surface temperature will become the new temperature at 0.19 km altitude. So the new surface temperature will be

$$T_{surf\text{-}new} = T_{surf0} - z * dT/dz = 288K - 0.19km*(-6.5K/km) = 288 + 1.25K$$

So according to this simple exercise, doubling CO_2 should cause the surface temperature to increase by about 1.25K.

This does not include the additional effect of the expected increase in water vapor as temperatures warm. Water is the dominant greenhouse gas in the Earth's atmosphere and its concentration depends on temperature. The positive water vapor feedback is thought to approximately double the effect of CO_2 alone, which, for the simple analysis done here indicates the surface temperature will increase by 2.5K.

"With CO2 at 550ppm (twice the pre-industrial, post-ice age level), average global temperatures would be between 2 and 4.5 C (3.6-8.1 F) higher than pre-industrial times, "with a best estimate of about 3 C (5.4 F)," says the (2008 IPCC) report."

So our simple analysis is in the same range as the big boys. This is why the predicted range of temperature increases has not changed much over the past 20 years.