Units, Units, Units and Coordinate systems…

See also Wallace&Hobbs, p. 467-468

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In general, we will use the *mks* set of units which is shorthand for meters, kilograms and seconds.

Length: meter (m), also kilometers (km)
Mass: kilograms (kg), also grams (g)
Time: seconds
Temperature: Kelvin, Celsius and Fahrenheit

Variables related to distance and time…

Position and coordinate systems

Position is the 3D location of a point and is usually written as $\vec{x}$ or $\vec{r}$ where the arrow indicates a vector. Sometimes bold is used to indicate a vector, ***x***.

- The zonal or east-west direction is indicated by an x with east being positive.
  - The zonal position may also be indicated by longitude in degrees. At the equator, 1^o of longitude is approximately 111 km. At higher latitudes, 1^o of longitude is approximately 111 km * cos(latitude).
- The meridional or north-south direction is indicated by a y with north being positive
  - The meridional position may also be indicated by latitude in degrees. 1^o of latitude is approximately 111 km.
- The vertical direction is indicated by a z with up being positive.
  - Pressure may also be used as a vertical coordinate because it simplifies some of the equations of motion. Pressure is indicated with a p. Note that pressure *decreases* as height *increases*.
  - When pressure is the vertical coordinate, height can still be used but height is a dependent variable indicating the height of a pressure surface like the 500 mb surface which varies with weather conditions.

Velocity: which is the rate of change in position with time (m/s) written as $\vec{v}$ or $\vec{u}$ or again with bold.

- Wind in the zonal direction is indicated by u with eastward motion being positive. This is also referred to as westerly motion meaning from the west.
- The meridional component of the wind is indicated by v with northward motion being positive. This is also referred to as southerly flow meaning from the south
- The vertical component of the wind is indicated with w with upward motion being positive.
- If the vertical component is pressure then the vertical component of the wind is indicated with ω.

Coordinate systems:
Regarding motion, there are two coordinate systems, Eulerian and Lagrangian

Acceleration: dv/dt in m/s^2.

Momentum is the mass of an object times the object's velocity which is a vector quantity, $m\vec{v}$

Angular momentum
$m\vec{v} \otimes \vec{r}$

Force is given by F = ma which says force equals mass times acceleration.
So a unit of force in mks is kg m/s^2 which is also known as a Newton.

Pressure is force per unit area.
So its natural units in mks must be Newtons per square meter = kg m/s^2 $/m^2$ = $kg/s^2/m$ which is known as a Pascal (Pa). 100 Pa = 1 millibar (mb) = 1 hectopascal (hPa).

Energy.
I find the easiest way to remember the units of energy is via kinetic energy which is ½ m v^2. So the standard units of energy in *mks* units must be kg m^2/s^2 which is known as a Joule (J).

Power
Power is the rate of change of energy or dE/dt. Its mks units are Joules per second which is known as a watt (W).

Density
Mass density is mass per unit volume, ρ or ρ_m in kg/m^3.
Energy density is energy per unit volume, ρ_E in J/m^3 which is also Pa. Note that pressure and energy density have the same units and are related but are not the same.
Number density is the number density, typically in number of objects per unit volume or in the case of molecules, moles per unit volume.

Flux
Flux is defined as a density times a velocity
Energy flux is $F_E = \rho_E \vec{v}$ which has units of $J/m^2/s = W/m^2$ which is power per unit area.

Flux divergence

The flux divergence is defined as $\nabla \bullet F = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z}$ which indicates how much the flux is spreading out spatially. A negative flux divergence means convergence, that is the flux, *F*, is converging or accumulating into a certain region or volume.

- For instance, when it is raining, atmospheric water vapor must be converging into the region where it is raining in order to supply the water that is condensing out

and falling to the ground. So the water vapor flux divergence would be negative in this region while it is raining.

Temperature

- Absolute temperature in Kelvin (K) is directly proportional to internal energy
- Celsius (C) = T(K) - 273.15
- Fahrenheit (F) = T(C) *9/5 + 32

Frequency

Temporal frequency is the rate of cycles of oscillation of something per second. The natural unit is 1 cycle per second = 1 Hertz (Hz).