Poleward heat transport by the atmospheric heat engine

Ref: Barry, L., G. C. Craig and J. Thuburn (2002), Poleward heat transport by the atmospheric heat engine, *Nature*, **415**, 774-777.

Here we summarize Barry et al. (2002) in *Nature* who apply aspects of both the Carnot cycle heat engine concept and diffusion to understand the energy transport by baroclinic eddies, what we know as large scale mid-latitude storm systems.

Abstract:

Atmospheric heat transport on Earth from the Equator to the poles is largely carried out by the mid-latitude storms. However, there is no satisfactory theory to describe this fundamental feature of the Earth's climate. Previous studies have characterized the poleward heat transport as a diffusion by eddies of specified horizontal length and velocity scales, but there is little agreement as to what those scales should be. Here we propose instead to regard the baroclinic zone—the zone of strong temperature gradients and active eddies—as a heat engine which generates eddy kinetic energy by transporting heat from a warmer to a colder region. This view leads to a new velocity scale, which we have tested along with previously proposed length and velocity scales, using numerical climate simulations in which the eddy properties have been varied by changing forcing and boundary conditions. The experiments show that the eddy velocity varies in accordance with the new scale, while the size of the eddies varies with the wellknown Rhines beta-scale. Our results not only give new insight into atmospheric eddy heat transport, but also allow simple estimates of the intensities of mid-latitude storms, which have hitherto only been possible with expensive general circulation models.

Utilizing the eddy correlation flux concept, one can write the meridional (poleward) transport of heat in an atmosphere as the vertical integral of

$$\overline{v'T'} = k \, \overline{v'^2}^{1/2} \, \overline{T'^2}^{1/2} = k \, \sigma_{v'} \sigma_{T'} \tag{1}$$

where k is the correlation between the meridional eddy velocity variations, v', and the eddy temperature perturbations, T'. Note that the correlation, k, is defined as

$$k = \frac{\overline{v'T'}}{\sigma_{v'}\sigma_{T'}} \tag{2}$$

So (1) follows from (2) which defines k.

Next, assume that the standard deviation of the temperature variations can be set equal to a displacement length, L_{disp} , in the meridional direction times the mean meridional temperature gradient.

$$\sigma_{T'} = -L_{disp} \frac{d\overline{T}}{dy} = -L_{disp} \overline{T_y}$$
(3)

Then we can use the diffusive flux equation to write

$$\overline{v'T'} = -D\overline{T_v} \tag{4}$$

where the meridional thermal diffusivity, *D*, is given as

$$D = L_{disp}\sigma_{v'} \tag{5}$$

The authors show that k varies from about 0.2 to 0,35 despite variations in the thermal flux of 2 orders of magnitude. So k appears to be approximately constant indicating the validity of the diffusive flux representation of the mid-latitude meridional heat flux. The trick is then to find valid expressions for the length and velocity scales, L_{disp} and σ_v , that represent the effective baroclinic eddy diffusivity

Concerning the length scale, the first point is that for a diffusive flux equation to apply, the length scale must be smaller than the meridional width of the baroclinic zone. We'll come back to this. For the length scale, they considered several possibilities.

1. Radius of deformation
$$L_D = \frac{NH}{f}$$
 (6)

where N is the buoyancy or Brunt-Vaisala frequency, H is the depth of the troposphere and f is the coriolis parameter = $2 \Omega \sin \phi$ where Ω is Earth's rotation rate and ϕ is the latitude.

2. Charney model fastest growing mode
$$L_{\gamma} = \frac{\frac{NH_s}{f}}{0.48 + 1.48\gamma}$$
 (7)

where H_s is the density scale height, β is the northward gradient of f, θ_z and θ_y are the vertical and meridional gradients of the potential temperature and

$$\gamma = \frac{\beta \theta_z H_s}{f \theta_y} \tag{8}$$

Other length scales are associated with cascading energy to larger horizontal scales (This cascading from smaller scales to larger scales is essentially a 2D energy cascade. 3D energy cascades from larger scales to smaller scales).

3. Energy cascade to the width of the baroclininc zone The largest scale to which this cascade could extend is the meridional width of the baroclinic zone itself, L_{zone} .

4. Energy cascade to the Rhines
$$\beta$$
-scale $L_{\beta} = \left(\frac{2\sigma_{\nu}}{\beta}\right)^{1/2}$ (9)

Next come the arguments for the eddy velocity scale in the baroclinic eddy thermal diffusivity.

1. Proportional to the zonal mean flow $\sigma_{v'} \propto \overline{u}$ (10)

2. equipartition of potential & kinetic
$$\sigma_{v'} \propto \overline{u} \frac{L_{disp}}{L_D}$$
 (11)

The authors propose another velocity scale derived from heat engine arguments where the heat engine extends over just the baroclinic zone rather than the entire atmosphere. Using the heat engine concept, the rate of generation and dissipation of eddy kinetic energy, ε , is given as

$$\varepsilon = e \frac{\delta T}{T_0} q \tag{12}$$

where *q* is the rate of energy transport out of the tropics calculated as the net diabatic (radiative plus surface flux) cooling per unit mass averaged over the baroclinic zone, $\delta T/T_0$ is the maximum possible thermodynamic efficiency of the heat engine where δT is the temperature difference between the regions where the heat enters and where it is extracted and *e* is the "utilization coefficient" which is the fraction of the kinetic energy used by the heat transporting eddies to the total generated kinetic energy.

The authors then argue that the velocity scale can only depend on the energy dissipation rate per unit mass, ε , and the length scale, L_{disp} , which based on the dimensions of these two variables (m²/s³ and m) results in

$$\sigma_{\nu'} \propto \left(\varepsilon L_{dsp}\right)^{1/3} \tag{13}$$

Plugging in (12) into (13) and assuming $L_{disp} \alpha L_{\beta}$ yields

$$\sigma_{\nu'} \propto \left(e \frac{\delta T}{T_0} q L_\beta \right)^{1/3} \tag{14}$$

Using (9) yields

$$\sigma_{v'} \propto \left(e \frac{\delta T}{T_0} q \left(\frac{2\sigma_{v'}}{\beta} \right)^{1/2} \right)^{1/3}$$
$$\sigma_{v'}^{5/2} \propto e \frac{\delta T}{T_0} q \left(\frac{2}{\beta} \right)^{1/2} = e \frac{a \overline{T_y}}{T_0} q \left(\frac{2}{\beta} \right)^{1/2}$$

where *a* is the radius of the Earth. So the velocity scale that results from this baroclinic heat engine argument is

$$\sigma_{v} \propto \left(e \frac{a \overline{T_y}}{T_0} q \right)^{2/5} \left(\frac{2}{\beta} \right)^{1/5}$$
(15)

The resulting baroclinic thermal diffusivity, D, is

$$D = L_{\beta}\sigma_{\nu} = \sigma_{\nu} \left(\frac{2\sigma_{\nu}}{\beta}\right)^{1/2} = \sigma_{\nu}^{3/2} \left(\frac{2}{\beta}\right)^{1/2} \propto \left(e\frac{a\overline{T_{y}}}{T_{0}}q\right)^{3/5} \left(\frac{2}{\beta}\right)^{4/5}$$
(16)

The heat flux is therefore

$$\overline{v'T'} = -D\overline{T_y} = -L_\beta \sigma_{v'} \overline{T_y} \propto -\left(e \frac{a\overline{T_y}}{T_0}q\right)^{3/5} \left(\frac{2}{\beta}\right)^{4/5} \overline{T_y} = -\left(e \frac{a}{T_0}q\right)^{3/5} \left(\frac{2}{\beta}\right)^{4/5} \overline{T_y}^{8/5}$$
(17)

Various combinations of these length and velocity scales used by various authors are summarized in the their Table 1.

Table 1 Summary of diffusivity parameters			
Theory	L	V'	D
Stone ⁴	L _D	π	kūL _D
Green ³	Lzone	$\overline{u}L_{zone}/L_{D}$	$k\overline{u}L_{zone}^2/L_D$
Haine and Marshall ⁷	Lzone	\overline{u}	kūL _{zone}
Held and Larichev ⁶	L _β	$\overline{u}\gamma^{-1}$	$k\overline{u}L_{\rm D}\gamma^{-2}$
Branscome ^{5,13}	$\frac{L_{\rm D}}{(0.48+1.48\gamma)}$	$\frac{\overline{u}}{(0.48+1.48\gamma)}$	$\frac{k\overline{u}L_{\rm D}}{(0.48+1.48\gamma)}$
This work	L _β	$\left(\frac{eaT_yq}{T_0}\right)^{2/5} \left(\frac{2}{\beta}\right)^{1/5}$	$\left(\frac{ea\overline{T}_{y}q}{T_{0}}\right)^{35} \left(\frac{2}{\beta}\right)^{45}$

 $L_{\rm D} = NH/f, L_{\beta} = (2|v'|/\beta)^{1/2} \text{ and } \gamma = \beta \theta_z H/(f\theta_v).$

They compare their length and velocity scales with the scales of baroclinic variability from a GCM. The comparison is in terms of scatter plots and correlation coefficients. It is quite clear that of the possibilities chosen, their length and velocity scales correlate significantly better with the GCM scales of variability than the other scales considered.



Figure 1 Predicted length scales versus diagnosed eddy length scale $L_{disp} = \overline{v'T'}/(k\overline{T}_y|v'|)$. The four predicted length scales are: **a**, $L_0 = NH/f$, the radius of deformation; **b**, L_{zone} , the size of the baroclinic zone; **c**, $L_\gamma = L_0/(0.48 + 1.48\gamma)$ where $\gamma = (\beta \theta_z H_s)/(f\theta_y)$; and **d**, $L_\beta = (2|v'|/\beta)^{1/2}$ is the Rhines β -scale. The four length scales have been normalized so that they are equal to L_{disp} for the Northern Hemisphere of the control climate. Symbol type represents the hemisphere, colour represents the type of experiment. NH, Northern Hemisphere; SH, Southern Hemisphere.



Figure 2 Predicted velocity scales versus diagnosed eddy velocity scale |v'|. The four predicted velocity scales are \overline{u} (**a**), $\overline{u}\gamma^{-1}$ (**b**), $\overline{u}/(0.48 + 1.48\gamma)$ (**c**) and $(ea\overline{T}_y q/T_0)^{2/5} (2/\beta)^{1/5}$ (**d**) (Symbols are defined in the text.). The four velocity scales have been normalized so that they are equal to |v'| for the Northern Hemisphere of the control climate. Symbol type and colour have same meaning as in Fig. 1. GCM, general circulation model.



Figure 3 Parametrized heat flux versus heat flux diagnosed directly from the GCM. The heat flux predicted by the new parametrization is $\overline{vT'}_{param} = k\overline{T}_{y}^{8/5}(eaq/T_{0})^{3/5}(2/\beta)^{4/5}$. The predicted flux has been normalized so that it is equal to the GCM flux for the Northern Hemisphere of the control climate. The symbol type and colour have the same meanings as in Fig. 1.

Clearly their new parameterization seems to work quite well in predicting the baroclinic meridional heat flux produced by the GCM indicating the heat engine and eddy diffusion concepts are applicable here. Such a relatively simple representation of such an important flux term leads to a better understanding of how this flux works and how it will behave as various variables are modified for instance in a changing climate.