Initiation of Rain in Clouds at Temperatures Above Freezing
(Largely following Rogers and Yau, 1989)

A challenge to precipitation physics is to explain how rain can develop in relatively little time as is observed, going from initial condensation and cloud formation to precipitation in as little as 20 minutes. From the previous lectures, we know that once supersaturation is sufficiently high to activate a CCN into an embryonic cloud particle, these particles can grow via diffusion into ~10 μ radius cloud particles in ~5 minutes. However, we have also shown that the time for a cloud droplet to grow to a 1 mm diameter sized raindrop via diffusion is more than a day which is far too slow relative to observations. Therefore there must be another process that is responsible for the observed transition from clouds with 100 droplets per cm^3 (10^8 per m^3) with a typical diameters of 20 μ to 1000 drops per m^3 with typical diameters of 1 mm.

In warm clouds, this process is one of collision and coalescence of cloud droplets into larger droplets which produces a 50 fold increase in drop size. This is an avalanche effect where relative motion between particles are slow initially and collisions therefore infrequent. But as a particle grows in size it falls more rapidly and collides more frequently causing it to grow more rapidly until it falls out of the cloud as a raindrop.

It turns out that to initiate this avalanche, droplets of 20 microns are needed to start colliding. The crosssections of smaller droplets are too small and they move too slowly and don't collide because their inertia is so small that they are readily deflected by the flow around the falling droplet. At 30 microns, coalescence is likely to be the dominant growth mechanism.

Note that the volume of a 1 mm (= 1000 microns) diameter droplet has $50^3 \sim 10^5$ times the volume of a typical cloud droplet and therefore may form from approximately 10^5 collisions. Only 1 droplet in 10^5 needs to be as big as 20 microns in radius or about 1/liter is needed to initiate this process.

Collision efficiency

Collision efficiency is the ratio of the actual collisions to the complete geometric sweep-outwhich depends on the collector drop size and sizes of the collected droplets

Consider a drop of radius, *R*, overtaking a droplet of smaller radius, *r*. Define the separation between the droplet centers as *x* which is called the impact parameter. There is a critical value, x_0, for a given *r* and *R* such that if the impact parameter is smaller than x_0 there is a collision and if $x > x_0$ there is no collision. The collision efficiency is defined as

$$E(R,r) = \frac{x_0^2}{(R+r)^2} \qquad (1)$$

The collision efficiency is equal to the fraction of those droplets with radius *r* in the path swept out by the collector drop that actually collide with it. Or *E* can be interpreted as the probability that a collision will occur with a droplet located at random in the swept volume.

A Short Course in Cloud Physics

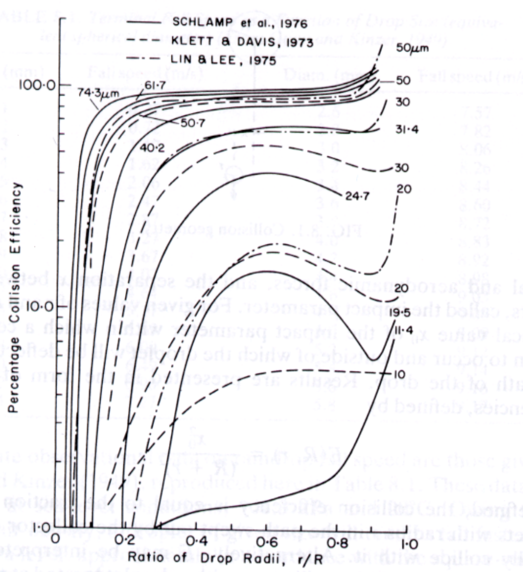

FIG. 8.2. Computed collision efficiencies for pairs of drops as a function of the ratio of their radii. Curves are labeled according to the radius R of the larger drop.

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TABLE 8.2. *Collision Efficiency* E *for Drops of Radius* R *Colliding with Droplets of Radius* r. *(Data for* R < 50 μm *adapted from Klett and Davis, 1973; for* 50 μm ≤ R ≤ 500 μm *from Beard and Ochs, 1984; for* R > 500 μm *from Mason, 1971)*

	r (μm)								
R (μm)	2	3	4	6	8	10	15	20	25
10	0.017	0.027	0.037	0.052	0.052				
20	*	0.016	0.027	0.060	0.12	0.17	0.17		
30	*	*	0.020	0.13	0.28	0.37	0.54	0.55	0.47
40	*	*	0.020	0.23	0.40	0.55	0.70	0.75	0.75
50	—	—	0.030	0.30	0.40	0.58	0.73	0.75	0.79
60	—	0.010	0.13	0.38	0.57	0.68	0.80	0.86	0.91
80	—	0.085	0.23	0.52	0.68	0.76	0.86	0.92	0.95
100	—	0.14	0.32	0.60	0.73	0.81	0.90	0.94	0.96
150	0.025	0.25	0.43	0.66	0.78	0.83	0.92	0.95	0.96
200	0.039	0.30	0.46	0.69	0.81	0.87	0.93	0.95	0.96
300	0.095	0.33	0.51	0.72	0.82	0.87	0.93	0.96	0.97
400	0.098	0.36	0.51	0.73	0.83	0.88	0.93	0.96	0.97
500	0.10	0.36	0.52	0.74	0.83	0.88	0.93	0.96	0.97
600	0.17	0.40	0.54	0.72	0.83	0.88	0.94	0.98	†
1000	0.15	0.37	0.52	0.74	0.82	0.88	0.94	0.98	†
1400	0.11	0.34	0.49	0.71	0.83	0.88	0.94	0.95	†
1800	0.08	0.29	0.45	0.68	0.80	0.86	0.96	0.94	†
2400	0.04	0.22	0.39	0.62	0.75	0.83	0.92	0.96	†
3000	0.02	0.16	0.33	0.55	0.71	0.81	0.90	0.94	†

—Collision efficiency less than 0.01.
*Value cannot be determined accurately from available data.
†Value close to one.

The collision efficiency is small for any ***small*** collector drop size. The collected droplets are then small and have little inertia and are easily deflected by the flow around the collector drop. The linear collision efficiency,

$$y_c = x_0/R \tag{2}$$

is also used in which case (1) is rewritten as

$$E = \frac{x_0^2}{(R+r)^2} = \frac{x_0^2}{R^2}\frac{R^2}{(R+r)^2} = y_c^2\frac{1}{\left(1+\frac{r}{R}\right)^2} = \frac{y_c^2}{(1+p)^2} \tag{3}$$

where $p = r/R$. Alternatively

$$E' = \frac{x_0^2}{R^2} \tag{4}$$

where $E' = E\,(1+p)^2$ and E' can therefore be > 1.

Growth equations

Given a drop of radius, R, falling at its terminal speed through a population of smaller droplets, it sweeps out a volume in unit time given by

$$\pi(R+r)^2\left[w(R)-w(r)\right] \tag{5}$$

From (5) the average number of droplets with radii between r and $r + dr$ collected in unit time is given by

$$\pi(R+r)^2\left[w(R)-w(r)\right]n(r)E(R,r)dr \tag{6}$$

where E here is the *collection* efficiency which equals the product of the collision efficiency and the coalescence efficiency.

The rate of increase of the collecting drop volume is obtained by integrating over all droplet sizes up to size R.

$$\frac{dV}{dt}=\int_0^R \frac{4}{3}\pi r^3\pi(R+r)^2\left[w(R)-w(r)\right]n(r)E(R,r)dr \tag{7}$$

The rate of increase of the droplet's radius is given by

$$\frac{dV}{dt}=4\pi R^2\frac{dR}{dt}=\int_0^R \frac{4}{3}\pi r^3\pi(R+r)^2\left[w(R)-w(r)\right]n(r)E(R,r)dr$$

$$\frac{dR}{dt}=\int_0^R \frac{1}{3}r^3\pi\left(\frac{R+r}{R}\right)^2\left[w(R)-w(r)\right]n(r)E(R,r)dr \tag{8}$$

Note that if we consider the case where the radius of the collector drop is much larger than the size of the drops being collected, then we can make the approximations, $R+r \sim R$ and $w(R) - w(r) \sim w(R)$ such that

$$\frac{dR}{dt}=\int_0^R \frac{1}{3}r^3\pi\, w(R)n(r)E(R,r)dr=\frac{w(R)}{4\rho_L}\int_0^R \frac{4}{3}\pi r^3\rho_L\, n(r)E(R,r)dr$$

$$\frac{dR}{dt}=\frac{w(R)}{4\rho_L}\int_0^R m_{drop}(r)\, n(r)E(R,r)dr=\frac{w(R)}{4\rho_L}M\overline{E} \tag{9}$$

where M is the cloud liquid water content, that is, the mass of condensed water per unit volume in the cloud and $\overline{E}$ is the effective average value of the collection efficiency for the droplet population.

The change in the drop size with altitude may be obtained from

$$\frac{dR}{dz}=\frac{dR}{dt}\frac{dt}{dz}=\frac{dR}{dt}\frac{1}{W-w(R)} \tag{10}$$

where W is the updraft velocity. Combining (9) and (10) gives

$$\frac{dR}{dz}=\frac{w(R)}{W-w(R)}\frac{M\overline{E}}{4\rho_L} \tag{11}$$

If the updraft speed is small in comparison to the terminal speed then (10) simplifies to

$$\frac{dR}{dz}=-\frac{M\overline{E}}{4\rho_L} \tag{12}$$

Note that these equations treat the cloud and the collection process as a continuum.

The problem with (11) is it predicts a growth rate that is too slow. The reason is (11) deals with the average growth. In reality it is the unusually large drops that initiate the precipitation not the average drops.

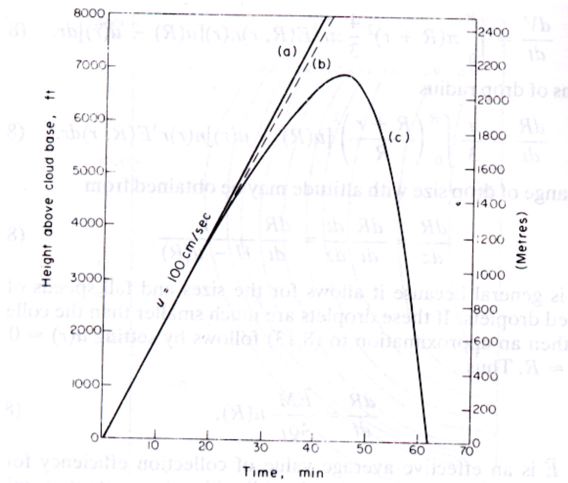


FIG. 8.4. Bowen's calculated trajectories of (a) the air, (b) cloud droplets, initially 10 μm in radius, and (c) drops which have initially twice the mass of the cloud droplets. Updraft speed 1 m/sec, cloud water content $M = 1$ g/m^3. (From Fletcher, 1962.)

INSERT FIGURES on trajectories

Continuum of droplet sizes and statistical growth

The concept that there is a distribution of larger drops overtaking a distribution of smaller droplets is skipped the physics about how this bimodal distribution of droplets came into being via the condensation-diffusion process. Presumably there is some process that makes the largest droplets resulting from this process become even larger. We need to develop a differential equation that describes how the droplet spectrum evolves in time. To do so, we define a *coagulation coefficient* that represents the probability that a drop of radius, *R*, will overtake and collide with a droplet of radius, *r*. The larger drop and smaller droplet sweep out volume per unit time given by

$$\pi R^2 w(R) \qquad \pi r^2 w(r)$$

The two volumes must overlap for the larger one to capture the smaller one such that the common volume that holds both droplets is

$$\pi(R+r)^2\left[w(R)-w(r)\right]$$

Including aerodynamic effects yields

$$K(R,r)=\pi(R+r)^2\left[w(R)-w(r)\right]E(R,r)$$

where $E(R,r)$ is the collision efficiency and $K(R,r)$ is called the coagulation coefficient or collection kernel. This can be rewritten in terms of droplet volumes, *V* and *v* corresponding to droplet radii, *R* and *r*,

$$H(V,v) = K\left[\left(\frac{3V}{4\pi}\right)^{1/3}, \left(\frac{3v}{4\pi}\right)^{1/3}\right]$$

Define n(v) as the number density of droplets with droplet volumes between *v* and *v+dv*. The total number of coalescences per unit time per unit volume experienced by drops within the size interval v to v+dv is

$$n(v)dv\int_0^\infty H(V,v)n(V)dV$$

in units of m^{-3} or equivalent (like cm^{-3}). Note that this includes collisions with droplets both larger and smaller than v. Note that with each coalescence, the number of droplets in the interval v to v+dv is reduced by 1. Note that the coalescence of two smaller droplets can result in a droplet of volume v. This is written as follows

$$\frac{1}{2}dv\int_0^v H(\delta,u)n(\delta)n(u)du$$

where $\delta = v-u$. The factor of ½ is to prevent counting a collision twice. Combining these last two equations yields the time rate of change of the number of droplets in the interval *v* to *v+dv*

$$\frac{\partial}{\partial t}n(v)dv = \frac{1}{2}dv\int_0^v H(\delta,u)n(\delta)n(u)du - n(v)dv\int_0^\infty H(V,v)n(V)dV$$

Integrating this equation does lead to a bimodal distribution where the largest droplets grow rapidly once they achieve a size around 20 microns in radius.

Rogers and Yau demonstrate this using examples from a study by Berry and Reinhardt (1974). The droplet spectrum is characterized by a probability density function $f(x)$ where $f(x)\,dx$ is the number of droplets per unit volume of air in the size interval (x, $x+dx$) and x is the droplet mass (not radius). The mass density function, $g(x)$, is defined by $x\,f(x)$. The droplet number density of concentration, N, and the liquid water content, L, are related to the probability density functions:

$$N = \int f(x)dx$$

$$L = \int g(x)dx$$

The mean mass of a droplet is given by

$$x_f = \frac{\int xf(x)dx}{\int f(x)dx} = \frac{L}{N}$$

A droplet whose size equals the mean droplet mass has a radius of

$$r_f = \left(\frac{3}{4}\pi\rho_L\right)^{1/3} x_f^{1/3}$$

The mean mass of the mass density function is defined by

$$x_g = \frac{\int xg(x)dx}{\int g(x)dx}$$

$$x_g = \frac{\int x^2 f(x)dx}{\int xf(x)dx}$$

and its equivalent radius is

$$r_g = \left(\frac{3}{4}\pi\rho_L\right)^{1/3} x_g^{1/3}$$

The following figure shows the evolution of a droplet spectrum due to coalescence beginning with $f(x)$ as a gamma distribution with $r_f = 12$ microns, $L = 1$ g/m^3 and $N = 166$ droplets per cm^3. The key points are the unimodal distribution is seen to evolve into a bimodal distribution and the liquid water in the original distribution moves into the large droplet distribution via the coalescence process.

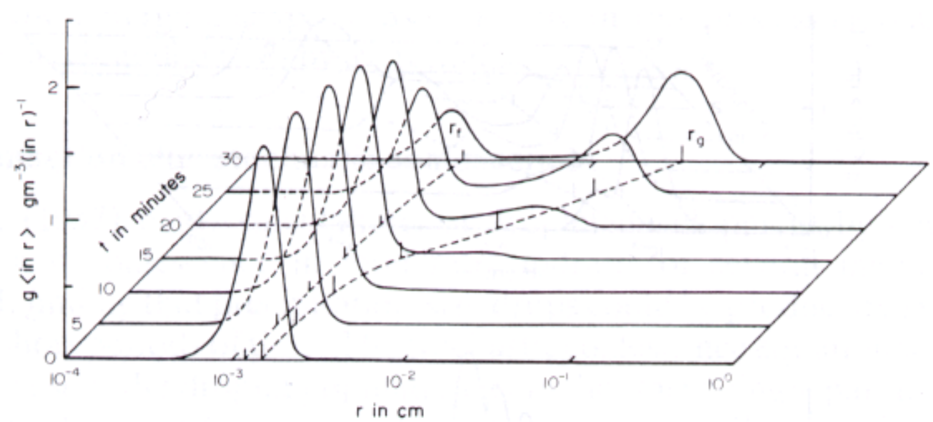


FIG. 8.10. Example of the development of a droplet spectrum by stochastic coalescence. (From Berry and Reinhardt, 1974b.)

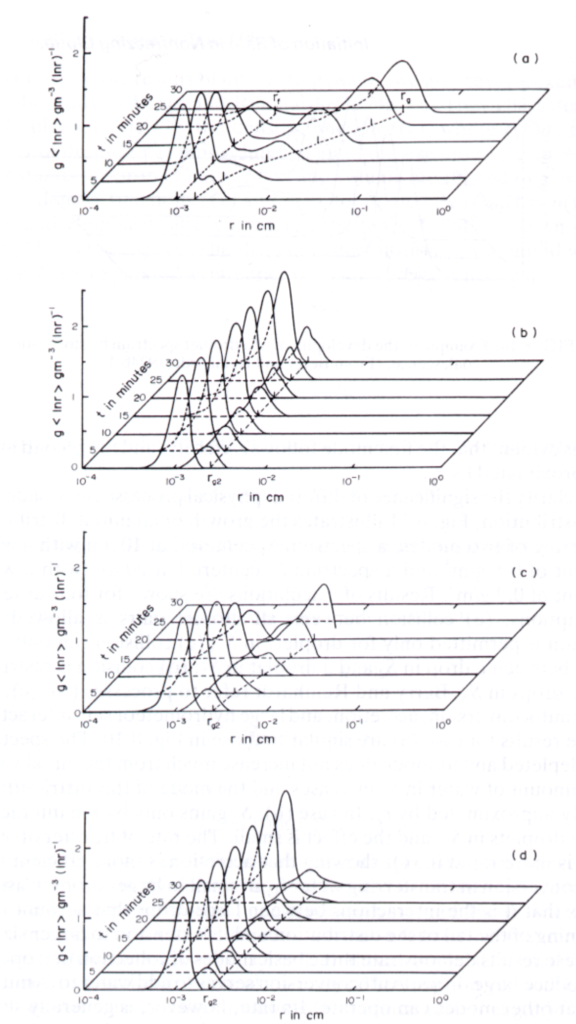


FIG. 8.11. Comparison of the evolution by coalescence of a drop spectrum, initially bimodal, by different collection processes. In each case the initial distribution consists of a spectrum of droplets S_1 centered at 10 μm radius and a spectrum S_2 of drops centered at 20 μm. (a) All collisions accounted for. (b) Only collisions between droplets in S_1 allowed. (c) Only collisions between a drop in S_2 and a droplet in S_1 allowed. (d) Only collisions between drops in S_2 considered. (Adapted from Berry and Reinhardt, 1974a.)

Berry and Reinhardt (1974) distinguish between 3 collision/coalescence processes in converting droplets from S_1 to S_2:

a. ***"autoconversion"***: (fig b) collisions between droplets in S_1
b. ***"accretion"***: (fig c) collisions between a droplet in S_1 and a droplet in S_2
c. ***"large hydrometeor self interaction"***: (fig d) collisions between a droplet in S_2 and a droplet in S_2.

Autoconversion is important in creating the S_2 distribution. Accretion is important in moving water from S_1 to S_2 and Large hydrometeor self interaction is important in the growth of the large droplets once accretion has moved a lot of the condensed water into the S_2 distribution

Aerosol indirect effects

The number of available aerosols can affect the number and size of cloud droplets and the conversion to precipitation droplets. There are two aerosol indirect effects. Increasing the number of aerosols increases the number of CCNs and therefore the number of cloud droplets. This leads to a reduction in the average size of each cloud droplet assuming a finite amount of water vapor is available for condensation into cloud droplets.

1. The first aerosol indirect effect is the surface area of the condensed water in the clouds increases because the ratio of surface area to volume of a sphere is inversely proportional to the cloud droplet radius and therefore is larger for smaller particles

$$\frac{A}{V} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r}$$

For a given *M*, the mass of condensed water per unit volume in the cloud, the number of cloud droplets for a monodispersive dropsize distribution is

$$n(r) = \frac{M}{\rho_L \frac{4}{3}\pi r^3}$$

and the total surface area of the cloud droplets is

$$n(r)4\pi r^2 = \frac{M}{\rho_L \frac{4}{3}\pi r^3} 4\pi r^2 = \frac{M}{\rho_L}\frac{3}{r}$$

Alternatively, if *n* is a given set by the number of aerosol CCNs, then we can solve for the cloud droplet radius

$$r = \left(\frac{3}{4}\frac{M}{\rho_L \pi n}\right)^{1/3}$$

and the cloud droplet surface area per unit volume is

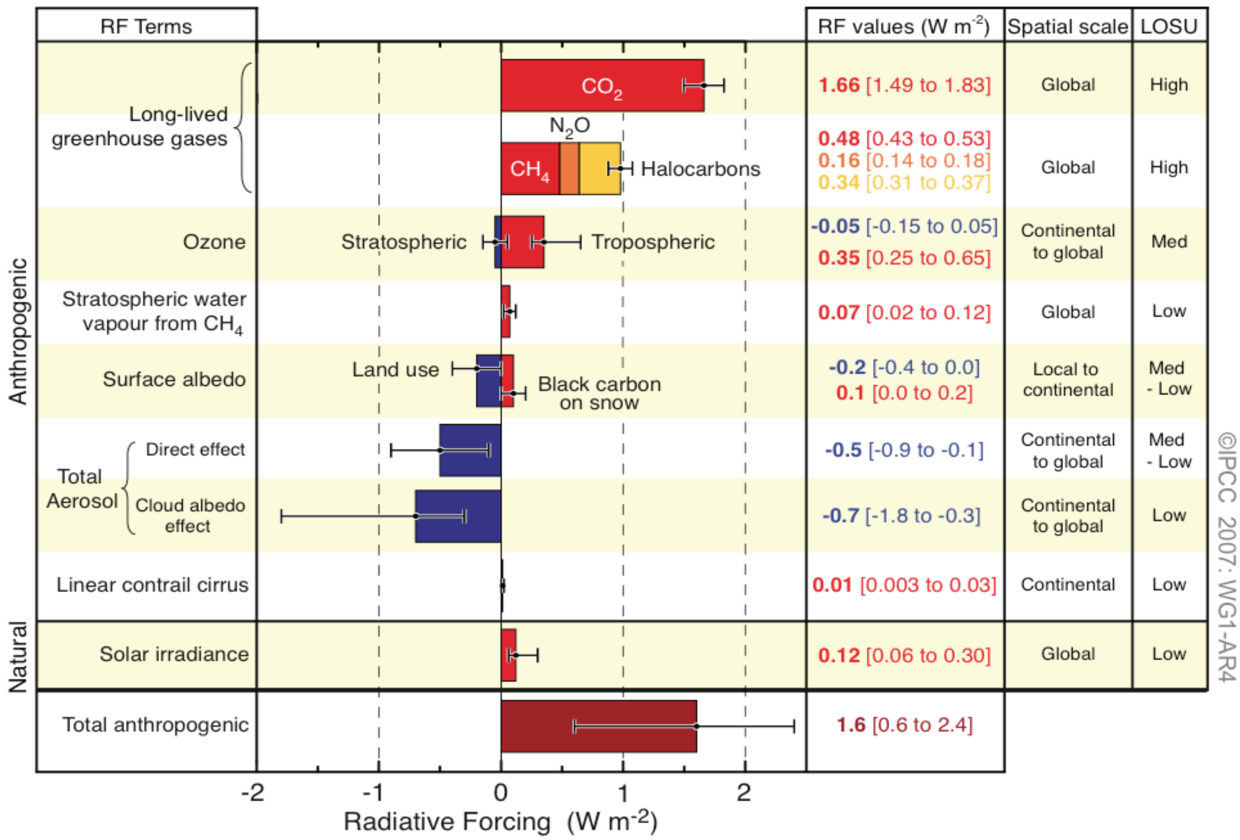
$$n4\pi r^2 = n4\pi\left(\frac{3}{4}\frac{M}{\rho_L \pi n}\right)^{2/3} = n^{1/3}4\pi\left(\frac{3}{4}\frac{M}{\rho_L \pi}\right)^{2/3} \propto n^{1/3}$$

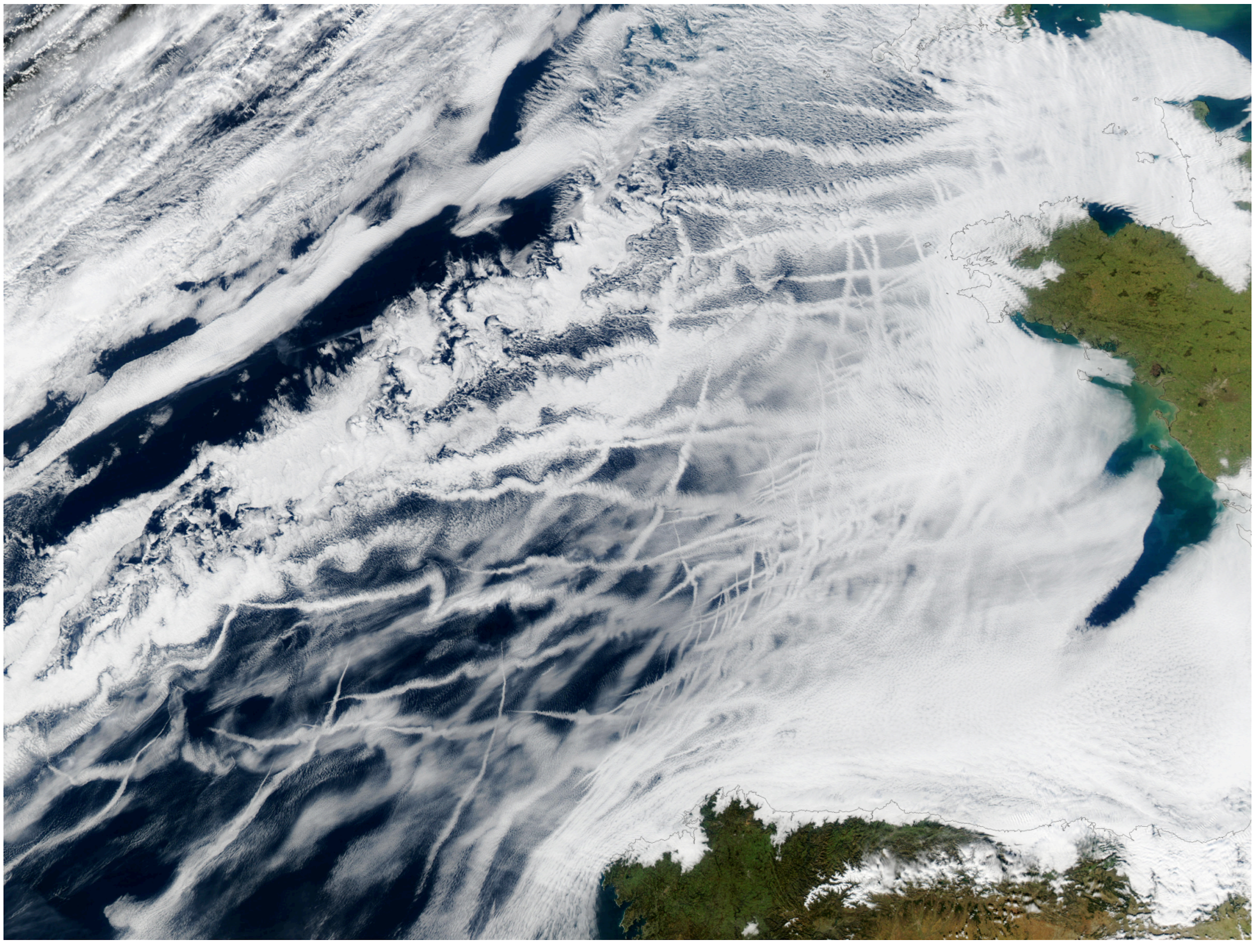
So the cloud surface area increases with the number of aerosol CCNs, assuming the cloud liquid water content remains the same. Doubling the number of aerosols increases the cloud surface area by 26%. According to Mie theory, reducing the cloud droplet size will reduce the forward scatter at visible wavelengths and therefore make the cloud droplets scatter more in directions other than forward which will make the cloud more reflective as well. This effect should make thin clouds become more reflective.

So the cloud formed with more aerosols is more reflective increasing the Earth's albedo. So the argument is anthropogenically created hygroscopic aerosols should reduce global warming.

2. The second indirect aerosol effect is that the increase in the number of aerosols will decrease the size of the cloud droplets reducing the number of droplets large enough to intiate autoconversion reducing the precipitation from the affected clouds. The decrease is because fewer droplets are able to reach the 20 micron range where autoconversion

begins which leads to droplet sizes large enough to precipitate out of the cloud. This is a change in the vertical heating profile and a reduction in precipitation from certain types of clouds. The overall impact of this 2nd indirect effect is not clear.





http://earthobservatory.nasa.gov/IOTD/view.php?id=3275